Pairing Heaps

	Fibonacci	Pairing
Insert	O(1)	O(1)
Remove min (or max)	O(n)	O(n)
Meld	O(1)	O(1)
Remove	O(n)	O(n)
Decrease key (or increase)	O(n)	O(1)

Actual Complexity

Pairing Heaps

	Fibonacci	Pairing
Insert	O(1)	O(log n)
Remove min (or max)	O(log n)	O(log n)
Meld	O(1)	O(log n)
Remove	O(log n)	O(log n)
Decrease key (or increase)	O(1)	O(log n)

Amortized Complexity

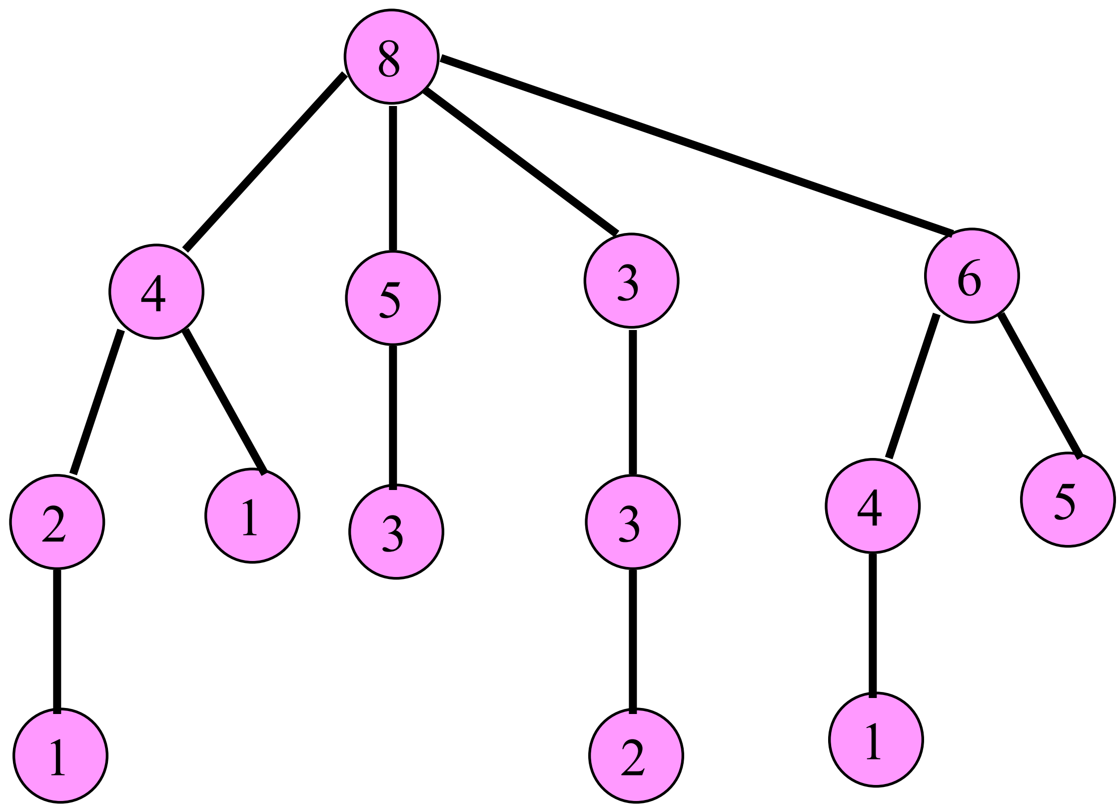
Pairing Heaps

- Experimental results suggest that pairing heaps are actually faster than Fibonacci heaps.
  - Simpler to implement.
  - Smaller runtime overheads.
  - Less space per node.

Definition

- A min (max) pairing heap is a min (max) tree in which operations are done in a specified manner.

Max Tree

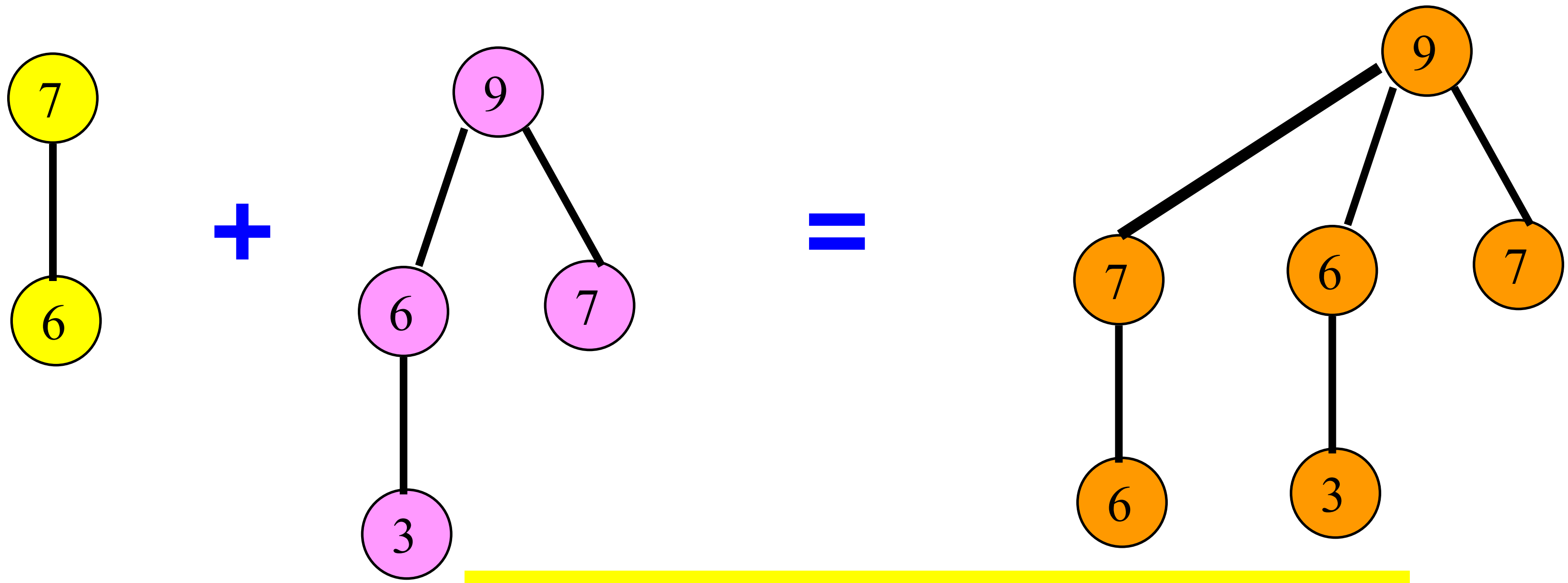


Node Structure

- Child
  - Pointer to first node of children list.
- Left and Right Sibling
  - Used for doubly linked linked list (not circular) of siblings.
  - Left pointer of first node is to parent.
  - x is first node in list iff x.left.child = x.
- Data
- Note: No Parent, Degree, or ChildCut fields.

Meld – Max Pairing Heap

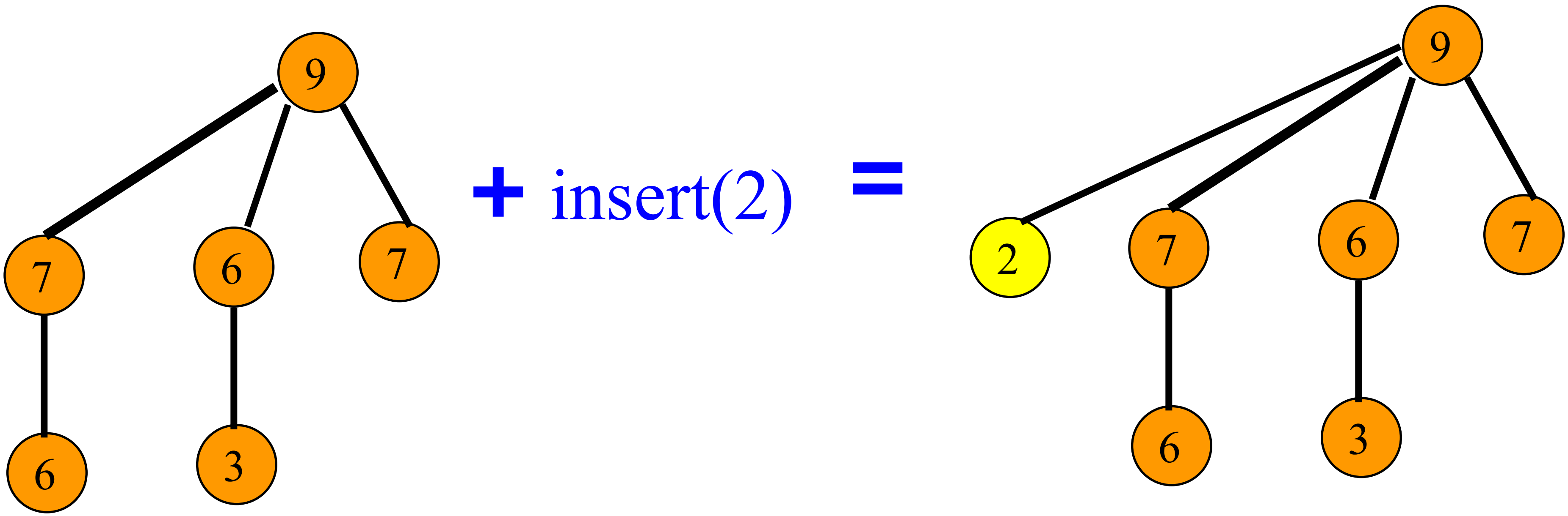
- Compare-Link Operation
  - Compare roots.
  - Tree with smaller root becomes leftmost subtree.



- Actual cost = O(1).

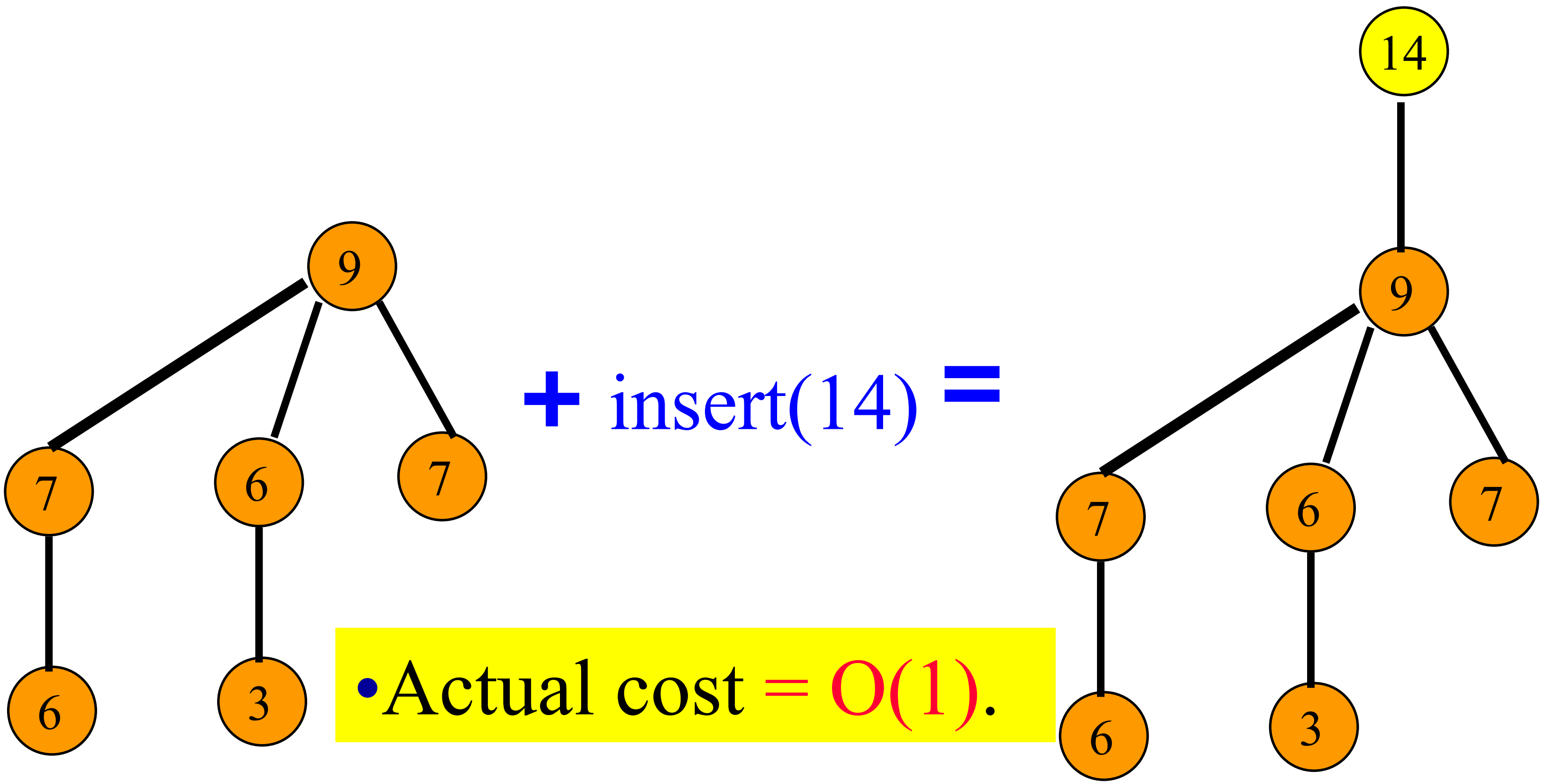
Insert

- Create 1-element max tree with new item and meld with existing max pairing heap.



Insert

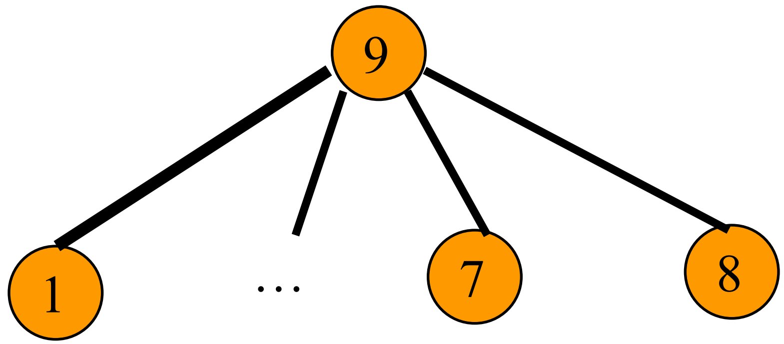
- Create 1-element max tree with new item and meld with existing max pairing heap.



- Actual cost = O(1).

Worst-Case Degree

- Insert 9, 8, 7, …, 1, in this order.



- Worst-case degree = n –1.

Worst-Case Height

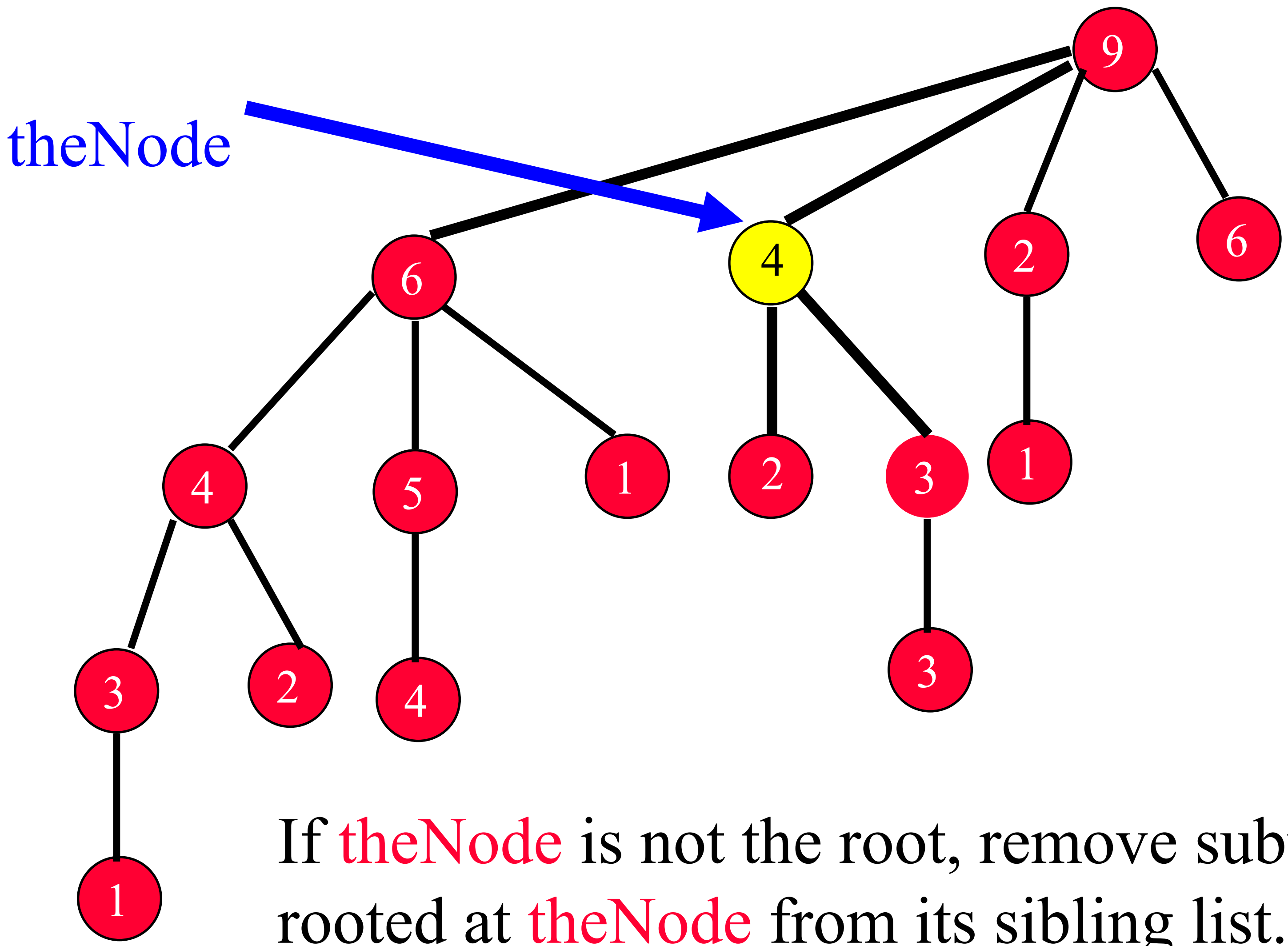
- Insert 1, 2, 3, …, n, in this order.

•Worst-case height = n.

IncreaseKey(theNode, theAmount)

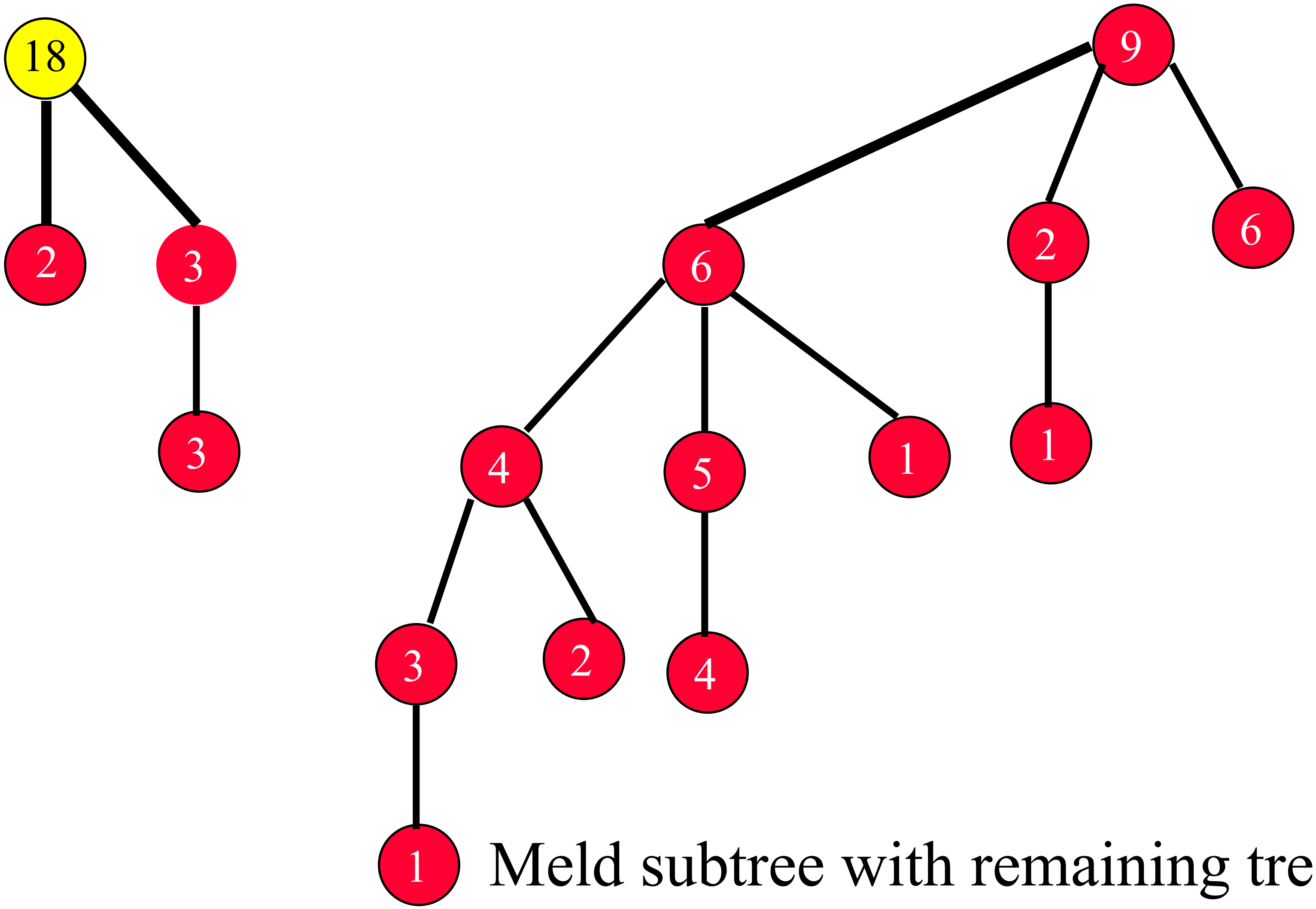
- Since nodes do not have parent fields, we cannot easily check whether the key in theNode becomes larger than that in its parent.
- So, detach theNode from sibling doubly-linked list and meld.

IncreaseKey(theNode, theAmount)



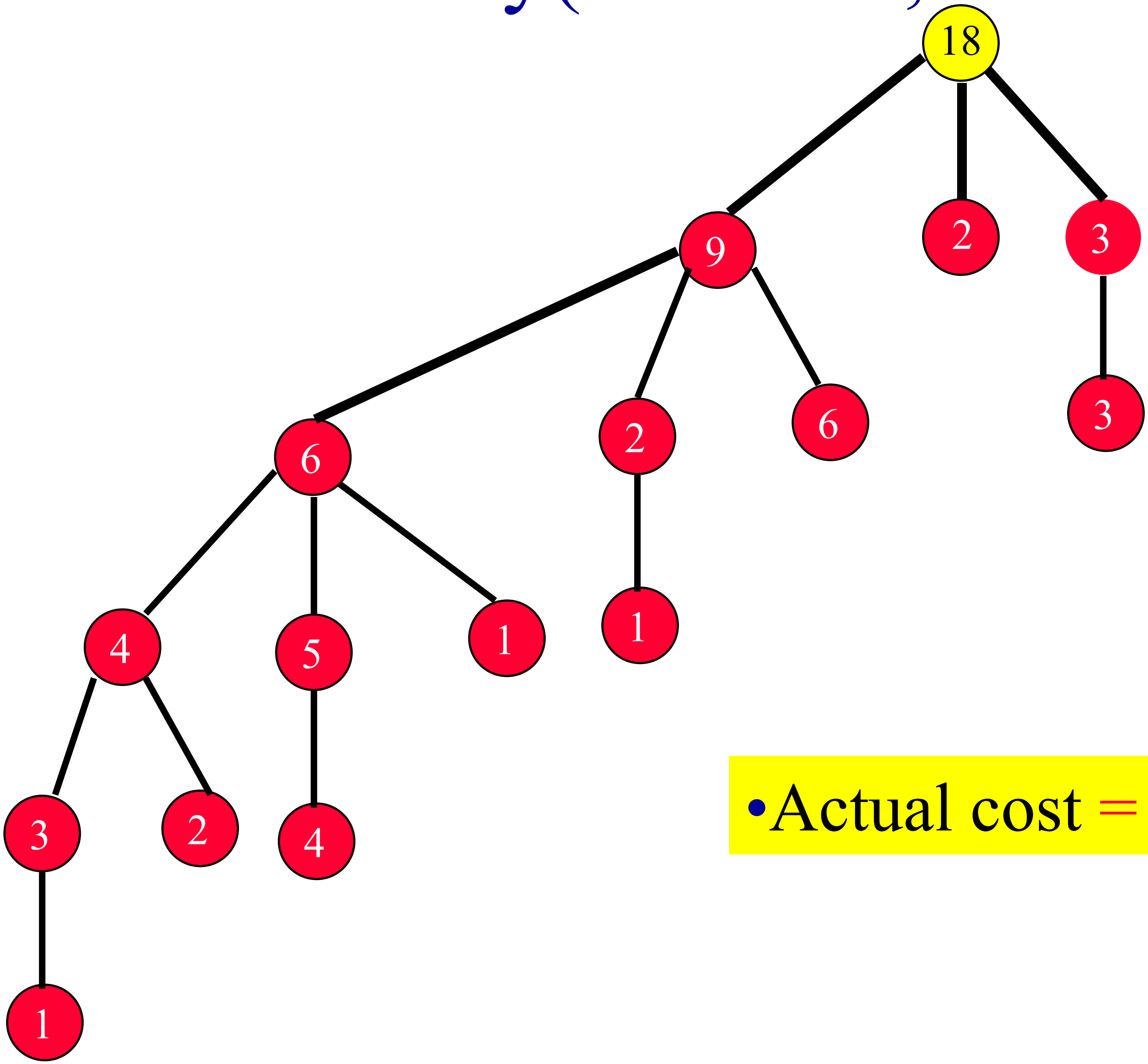
If theNode is not the root, remove subtree rooted at theNode from its sibling list.

IncreaseKey(theNode, theAmount)



Meld subtree with remaining tree.

IncreaseKey(theNode, theAmount)



- Actual cost = O(1).

Remove Max

- If empty => fail.
- Otherwise, remove tree root and meld subtrees into a single max tree.
- How to meld subtrees?
  - Good way => O(log n) amortized complexity for remove max.
  - Bad way => O(n) amortized complexity.

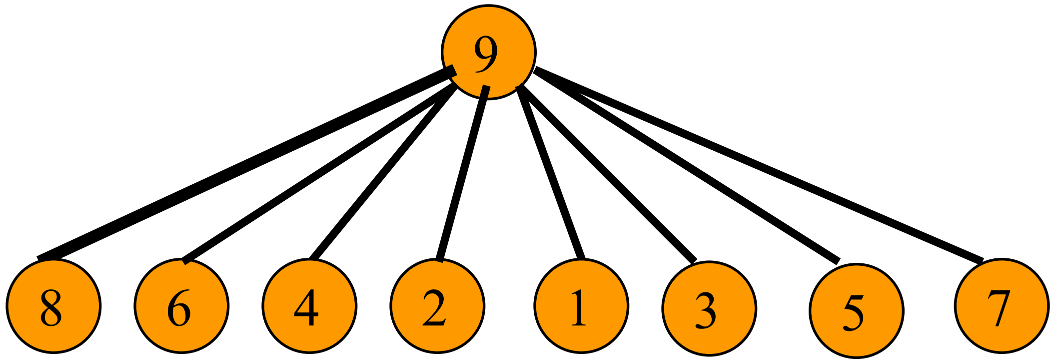
Bad Way To Meld Subtrees

- currentTree = first subtree.
- for (each of the remaining trees)

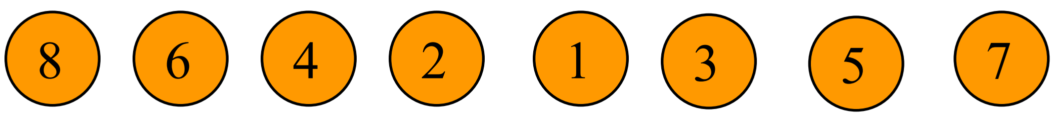
  currentTree = compareLink(currentTree,

  nextTree);

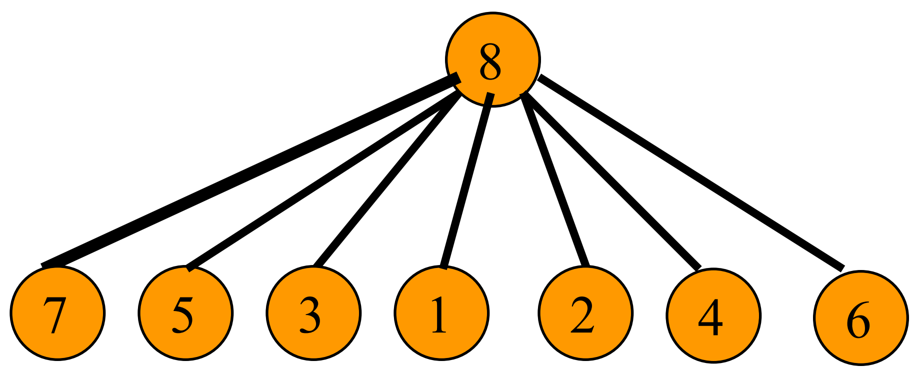
Example



- Remove max.



- Meld into one tree.



Example

- Actual cost of insert is 1.
- Actual cost of remove max is degree of root.
- n/2 inserts (9, 7, 5, 3, 1, 2, 4, 6, 8) followed by n/2 remove maxs.
  - Cost of inserts is n/2.
  - Cost of remove maxs is $1 + 2 + \ldots + n/2 - 1 = \Theta(n^2)$.
  - If amortized cost of an insert is $O(1)$, amortized cost of a remove max must be $\Theta(n)$.

Good Ways To Meld Subtrees

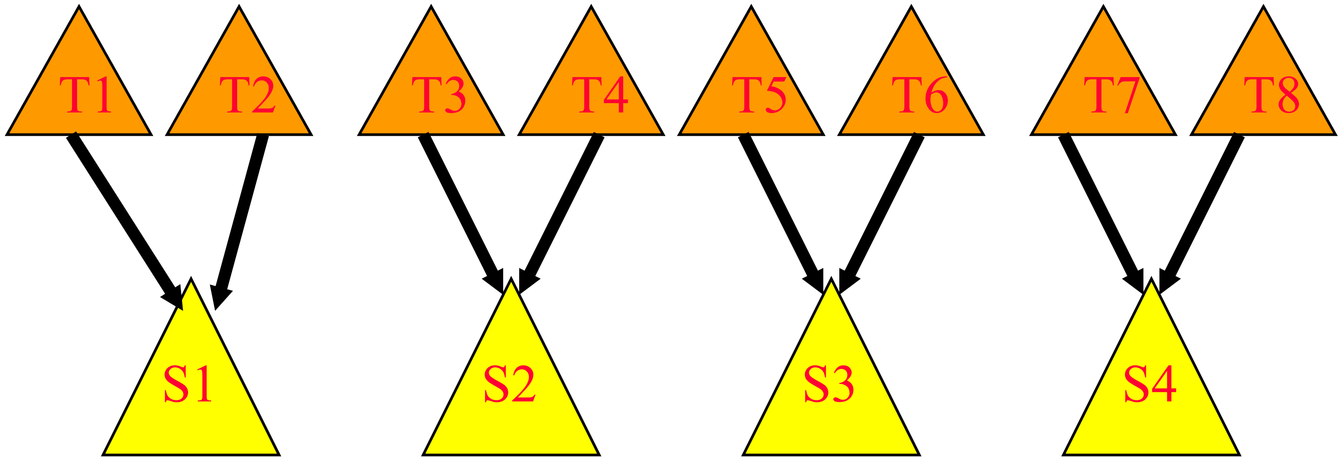
- Two-pass scheme.
- Multipass scheme.
- Both have same asymptotic complexity.
- Two-pass scheme gives better observed performance.

Two-Pass Scheme

- Pass 1.
  - Examine subtrees from left to right.
  - Meld pairs of subtrees, reducing the number of subtrees to half the original number.
  - If # subtrees was odd, meld remaining original subtree with last newly generated subtree.
- Pass 2.
  - Start with rightmost subtree of Pass 1. Call this the working tree.
  - Meld remaining subtrees, one at a time, from right to left, into the working tree.

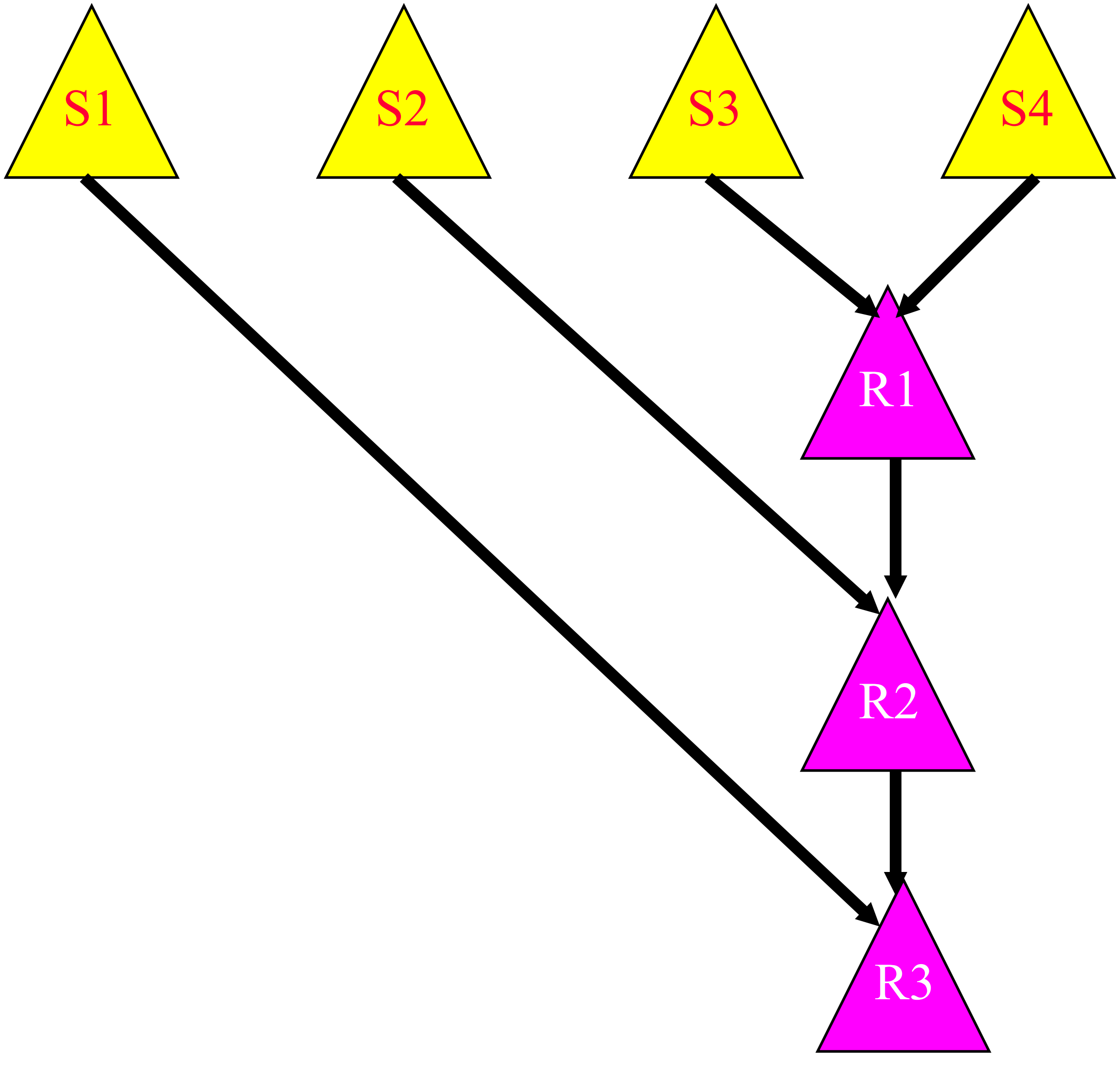
Two-Pass Scheme – Example

Pass 1



Two-Pass Scheme – Example

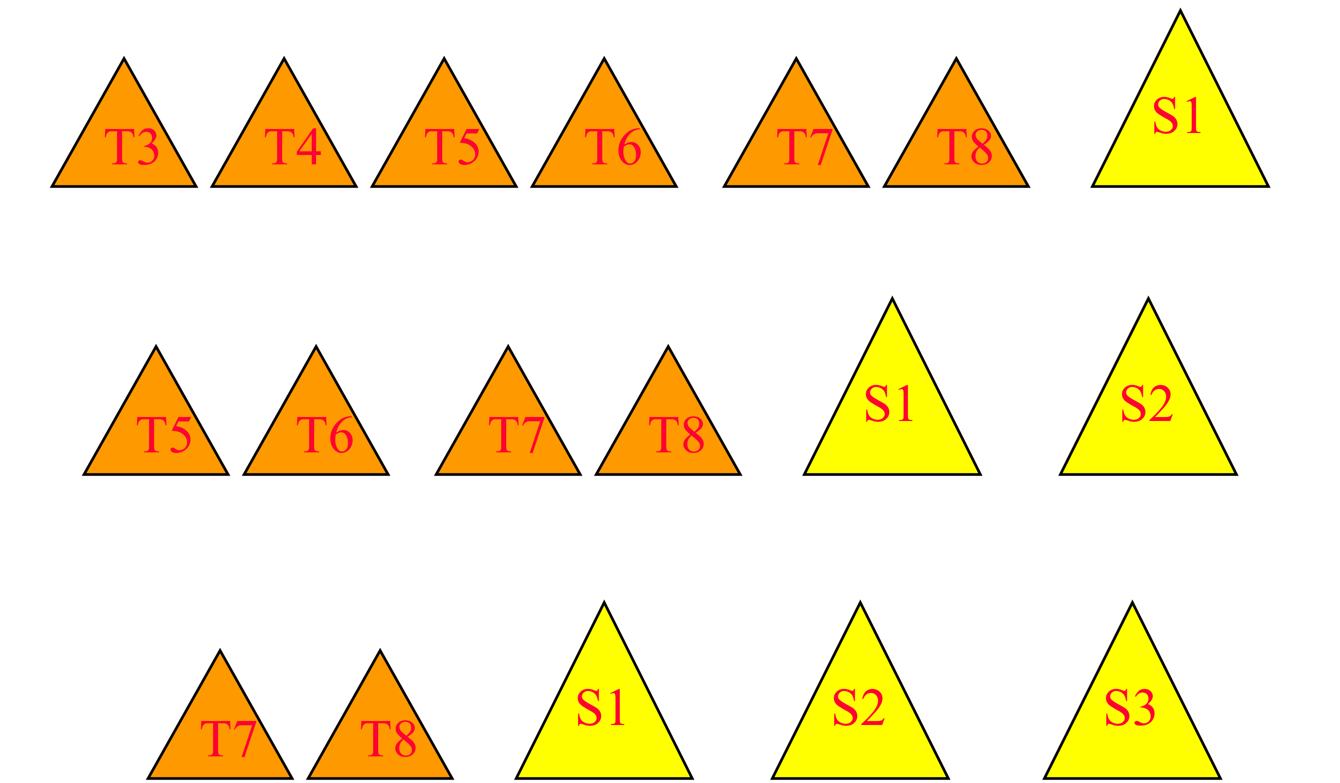
Pass 2



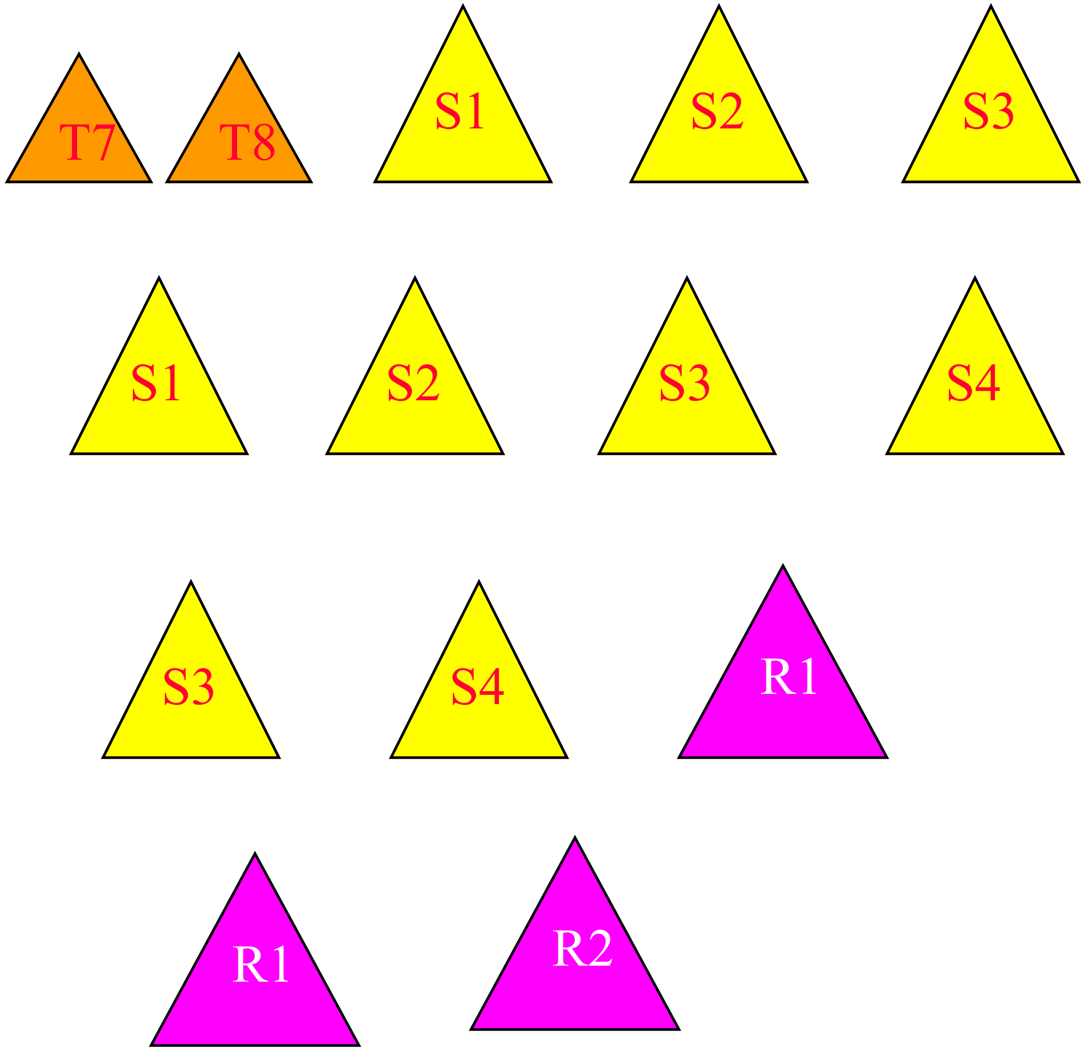
Multipass Scheme

- Place the subtrees into a FIFO queue.
- Repeat until 1 tree remains.
  - Remove 2 subtrees from the queue.
  - Meld them.
  - Put the resulting tree onto the queue.

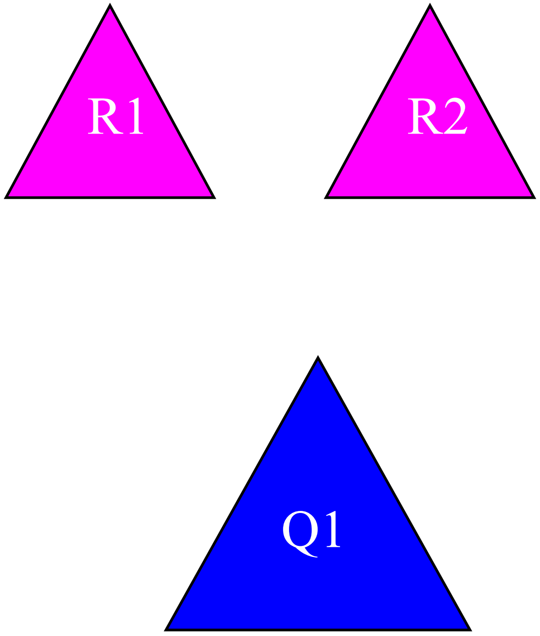
Multipass Scheme – Example



Multipass Scheme--Example



Multipass Scheme--Example

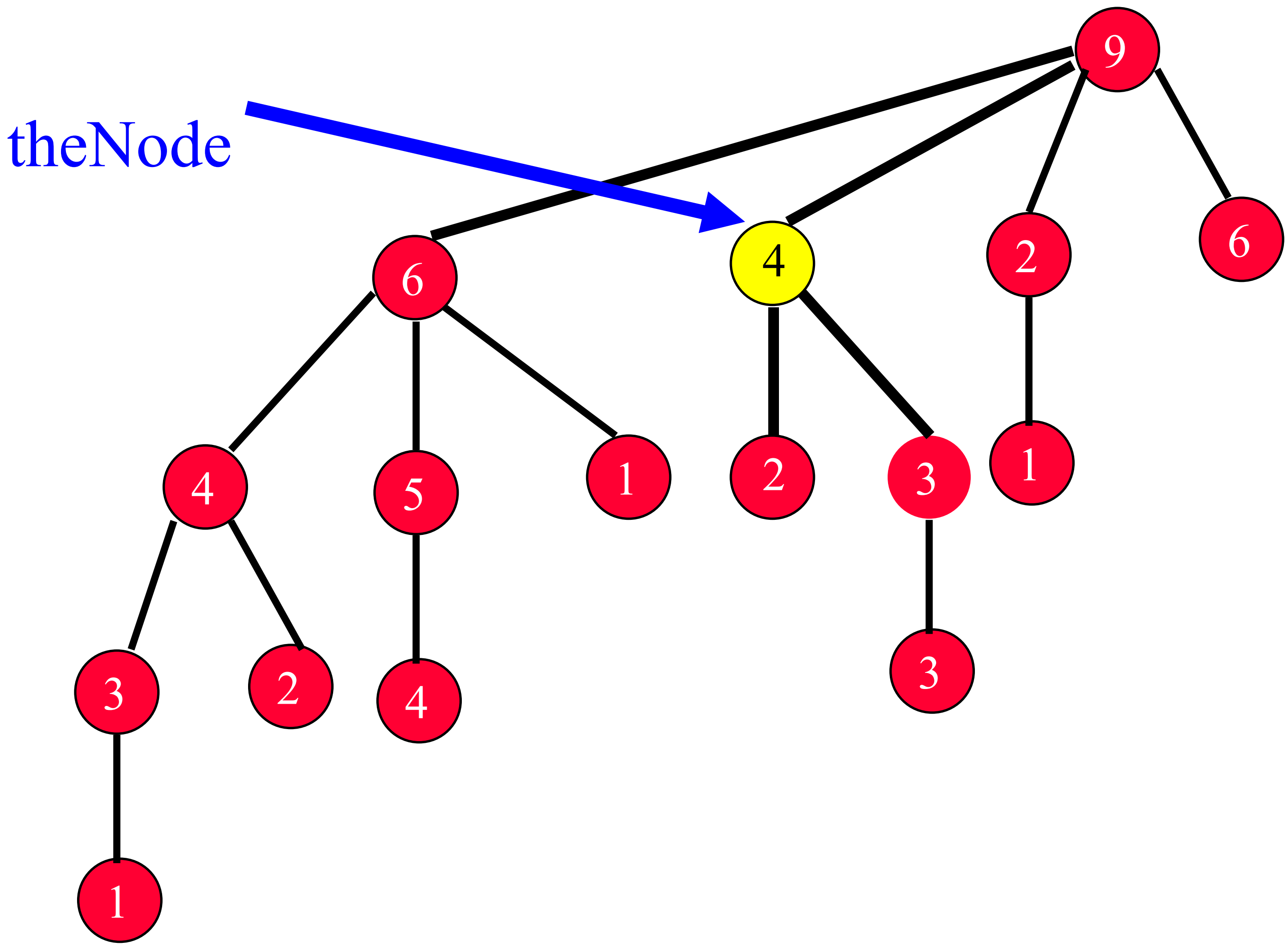


- Actual cost = O(n).

Remove Nonroot Element

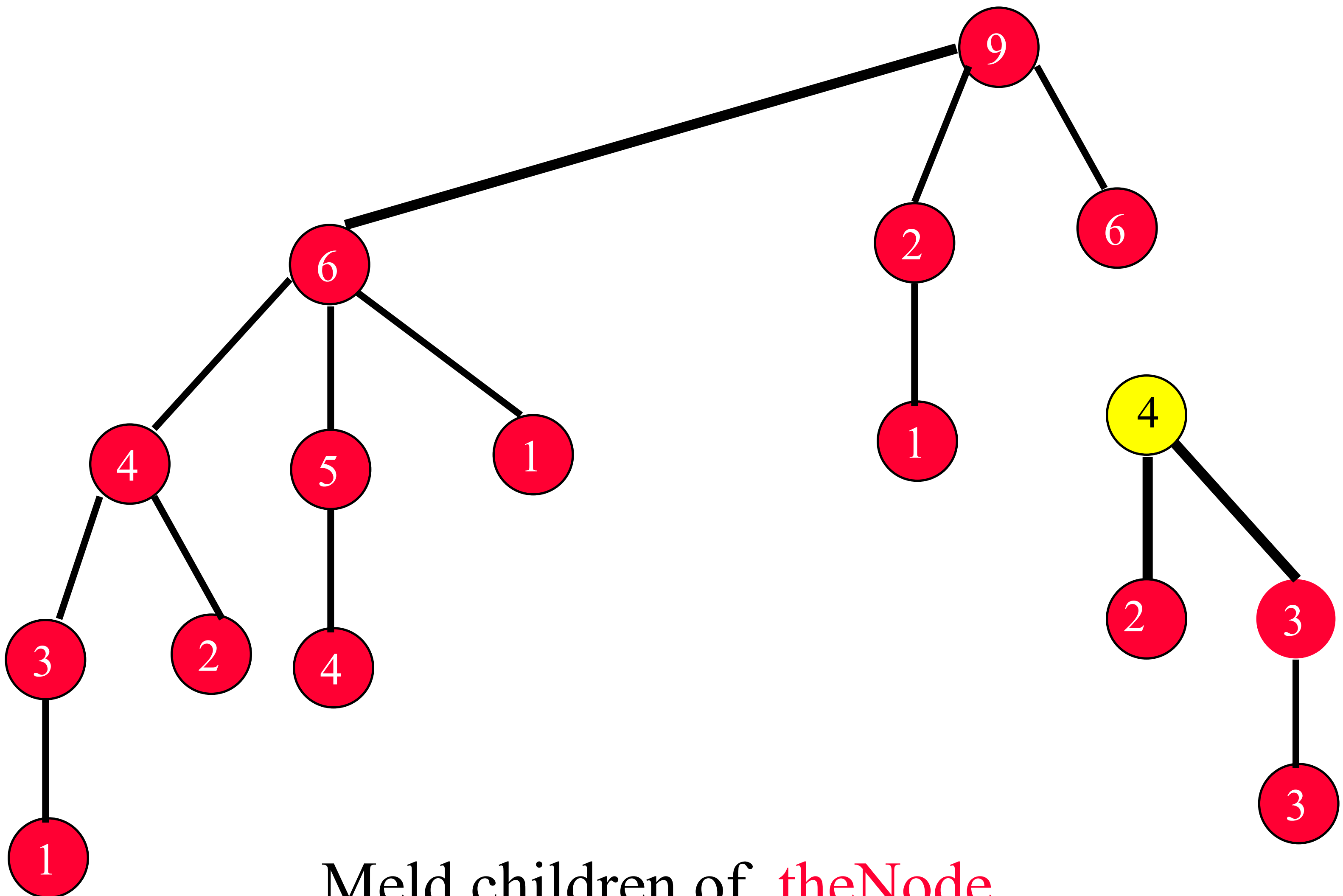
- Remove theNode from its sibling list.
- Meld children of theNode using either 2-pass or multipass scheme.
- Meld resulting tree with what's left of original tree.

Remove(theNode)



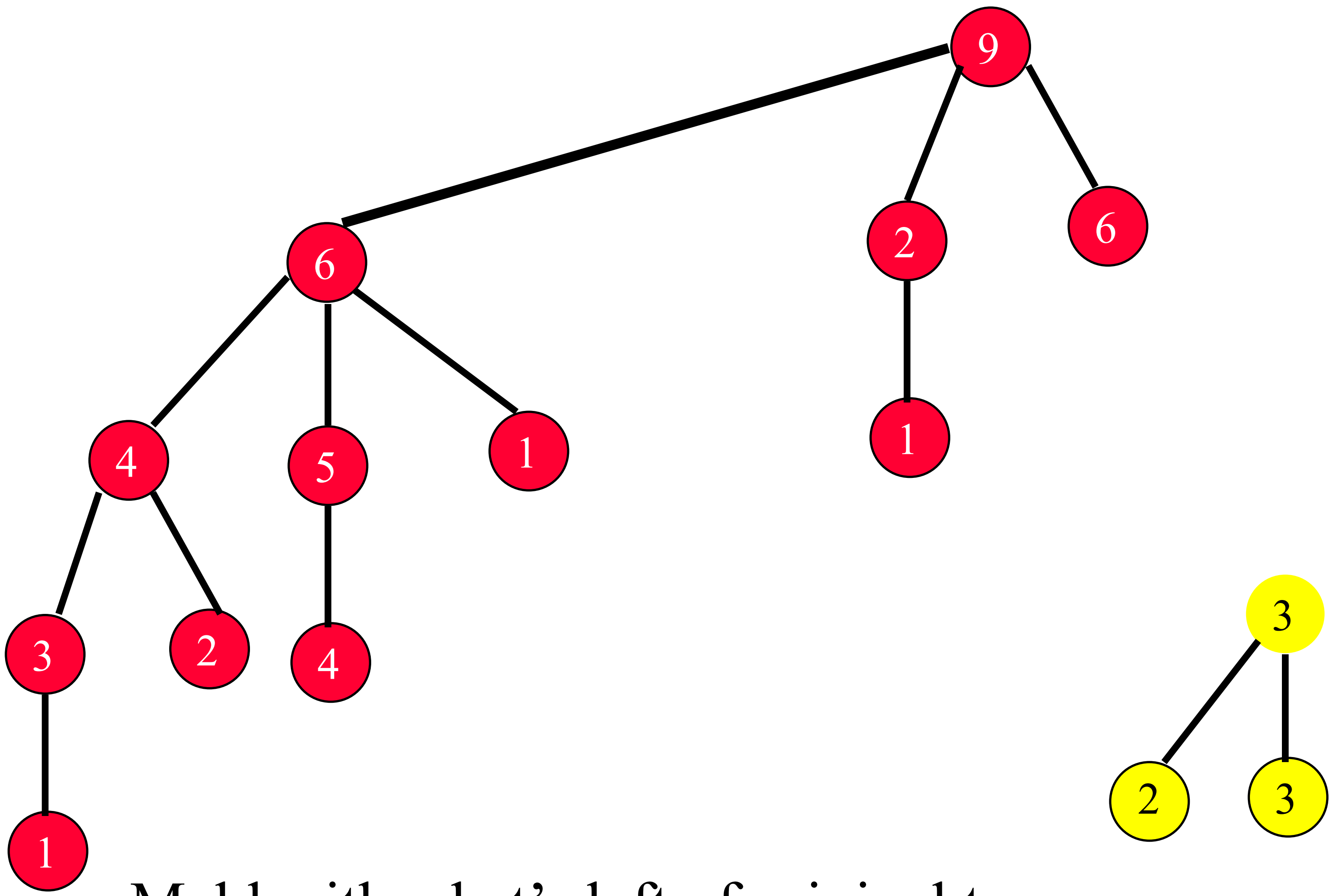
Remove theNode from its doubly-linked sibling list.

Remove(theNode)



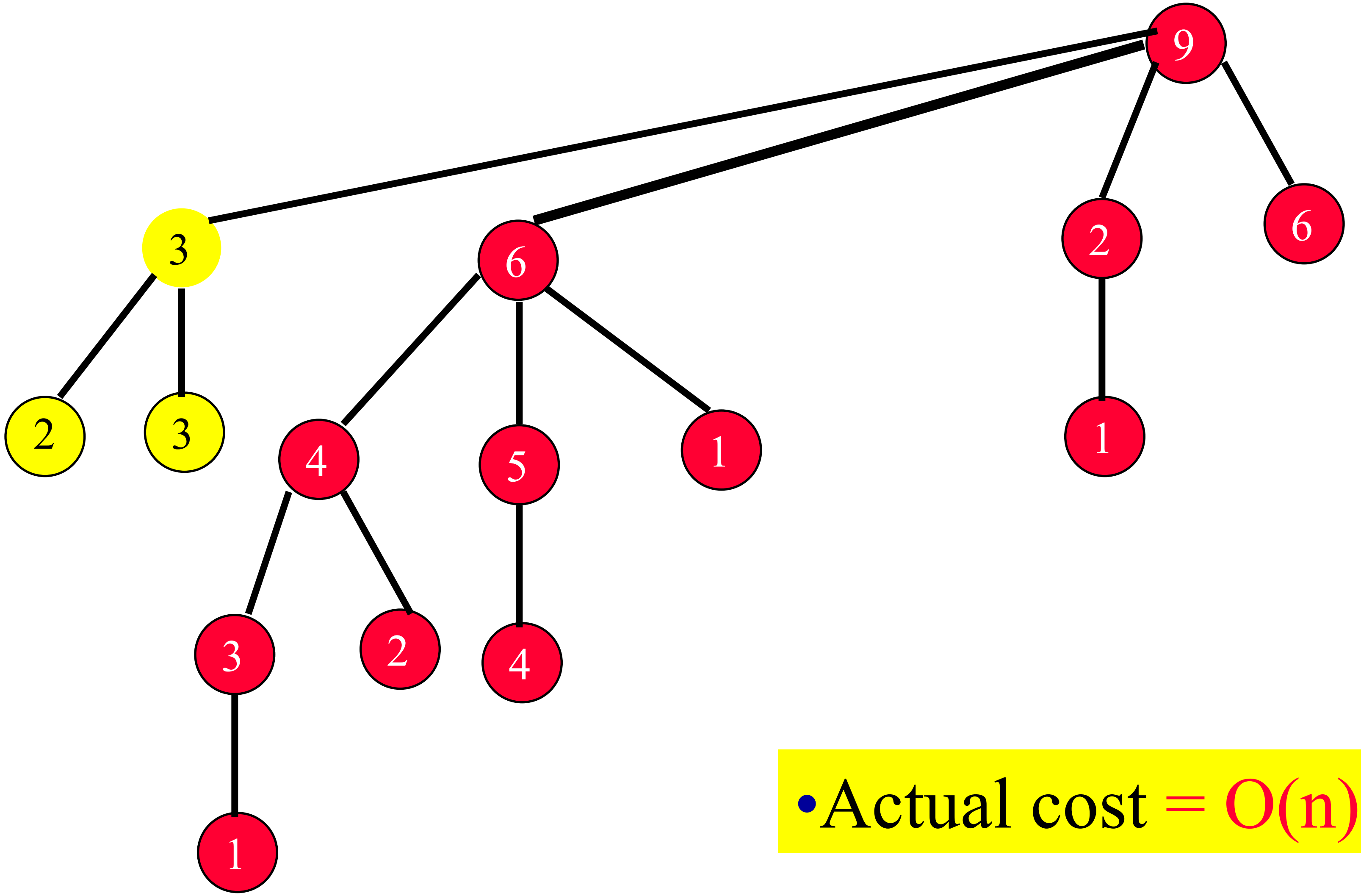
Meld children of theNode.

Remove(theNode)



Meld with what's left of original tree.

Remove(theNode)



- Actual cost = O(n).