Controllable uncertain opinion diffusion under confidence bound and unpredicted diffusion probability

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HIGHLIGHTS

• The diffusion probability is modified according to the distance between the opinions of interacted nodes.
• An optimization problem and suitable algorithm to show how to control the uncertain diffusion process.
• The performances of traditional influence maximization algorithms are often worse than that of random selection in this optimization problem.
• Reason of that the traditional influence maximization algorithms are not applicable to this problem.

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ABSTRACT

The issues of modeling and analyzing diffusion in social networks have been extensively studied in the last few decades. Recently, many studies focus on uncertain diffusion process. The uncertainty of diffusion process means that the diffusion probability is unpredicted because of some complex factors. For instance, the variety of individuals’ opinions is an important factor that can cause uncertainty of diffusion probability. In detail, the difference between opinions can influence the diffusion probability, and then the evolution of opinions will cause the uncertainty of diffusion probability. It is known that controlling the diffusion process is important in the context of viral marketing and political propaganda. However, previous methods are hardly feasible to control the uncertain diffusion process of individual opinion. In this paper, we present suitable strategy to control this diffusion process based on the approximate estimation of the uncertain factors. We formulate a model in which the diffusion probability is influenced by the distance between opinions, and briefly discuss the properties of the diffusion model. Then, we present an optimization problem at the background of voting to show how to control this uncertain diffusion process. In detail, it is assumed that each individual can choose one of the two candidates or abstention based on his/her opinion. Then, we present strategy to set suitable initiators and their opinions so that the advantage of one candidate will be maximized at the end of diffusion. The results show that traditional influence maximization algorithms are not applicable to this problem, and our algorithm can achieve expected performance.

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1. Introduction

Previous studies have modeled and analyzed various diffusion processes in social networks [1–6]. Generally, each node in social network is assumed to associate with the binary states-active or inactive. The state of a node may be switched

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based on some rules in the diffusion process. Among the models with different rules, the independent cascade model [3,7] is one of the models that are used the most widely in diffusion analyses. In the independent cascade model, the state of a node is switched based on a probability, and each activation process is independent of others. In various cases, the diffusion probability may be constant [7] or variable [8,9].

Recently, more and more studies have focused on the uncertain diffusion process. The uncertainty of diffusion process means that the diffusion probability is unpredicted because of some complex factors. Previous study has shown that the variety of individuals’ opinions is an important factor that can cause uncertainty of diffusion probability [5]. In detail, when someone receives information from others who hold similar opinions with him/her, he/she will be more likely to perform the dissemination behavior [10–12]. Besides, it has been known that opinions can evolve as individuals are reported to assimilate the viewpoints of interacted people [10,13]. Therefore, the diffusion probability becomes uncertain because of the opinion evolution.

Individuals express their opinions in online social networks frequently, such as the tastes in music or movie [10,13], the comments about innovations of product [14,15], and attitudes to the advertisements of candidate [16]. Therefore, these may be various diffusion processes of these opinions in social networks. It means that controlling these uncertain opinion diffusion processes is very important in the context of viral marketing [17] and political propaganda [16]. However, previous studies of opinion [13,18–20] just focus on how opinions evolve to the consensus or the polarization states. These studies usually discuss the influence of network structure, nodal degree or the evolution rules of opinion. As these factors are almost uncontrollable, it is costly or hardly feasible to control the diffusion process based on the conclusions obtained in these studies.

In this paper, in order to control the uncertain opinion diffusion, we discuss this diffusion process and develop feasible strategy to control this diffusion process based on the approximate estimation of the uncertain factors.

1.1. Motivation and contributions

In this paper, we raise the issue of controlling the uncertain opinion diffusion process. In order to show how to control the diffusion process, we present a diffusion model in which the diffusion probability is modified according to the distance between the opinions of interacted nodes. Moreover, each node which is exposed to the information can adjust its opinion depending on the assimilation of others’ opinions.

In detail, our contributions can be divided into two aspects:

- First, based on theoretical analyses and simulations, we briefly discuss the properties of the diffusion model in which the diffusion probability is influenced by the distance between opinions. Some conclusions are obtained and helpful in developing strategy to control the diffusion process.
- Second, we present an optimization problem to show how to control the uncertain diffusion process. In detail, we model a voting mechanism in which each node can choose one of the two candidates or abstention based on its opinion. Then, we present suitable strategy to maximize the advantage of one candidate by setting suitable initiators and their opinions in the diffusion process.

This paper is organized as follows. In Section 2, we briefly introduce the related work. Section 3 presents our diffusion model, including the definition of individual opinion, the modification of diffusion probability, and the evolution rule of opinions. In Section 4, we show the properties of the diffusion model, such as how wide the initiator’s opinion can spread under different initial distributions of opinions. Then, an optimization problem is provided in Section 5. Finally, we conclude this paper in Section 6.

2. Related works

2.1. Uncertain diffusion process

Many studies have focused on the diffusion process in which the diffusion probability is uncertain. For instance, structural uncertainty [3,21–23], input of multiple entities [8,9,24,25] and the variety of individuals’ opinions [5] are three factors that can cause the uncertainty of diffusion probability in previous studies. Adiga et al. [3] raise the issue of diffusion process interfered by structural uncertainty. The structural uncertainty is generally modeled by rewiring edges in the networks [3,21]. Additionally, Adiga et al. [3] mathematically investigate the sensitivity of diffusion process to the structural uncertainty. They also predicted the scales of information spreading affected by the different extents of perturbation. On the other hand, the spreading of multiple entities in social networks is another interference factor to diffusion probability. For instance, two competing diseases are introduced, and being infected by one disease gives a node partial or complete immunity to another [24]. By analyzing the empirical data of multiple contagions in Twitter, Myers and Leskovec reported the great effects of interaction between cooperating and competing information on spreading probabilities [25]. In addition, the variety of individuals’ opinions is also an important factor that can cause uncertainty of diffusion probability [5]. The distance between the opinions of interacted nodes can influence the diffusion probability, and the evolution of opinions
makes the diffusion probability become uncertain. However, this study [5] just presents a diffusion model and analyzes its properties. In this paper, we further present an optimization problem to show how to control the uncertain opinion diffusion process.

2.2. The issue of opinion

The issue of opinion has been studied from many disciplines [13,18–20]. Three models are used widely to analyze the opinion formation. They are averaging model [18], bounded confidence model [19] and voter model [26]. In the averaging model, the nodes update their opinions by averaging their opinions with the mean of their neighboring opinions. Besides, in many studies based on this model, different weights are attached to opinions [18]. Therefore, the result of opinion evolution heavily depends on these weights. In the bounded confidence model, each node has a confidence bound. If the distance between a node's opinion and another opinion is larger than the confidence bound, the node will not care this opinion at all [19]. In the voter model, at each time step, the nodes randomly choose a neighbor and adopt its opinion [26]. The opinions in the voter model are usually represented by a binary value, and the condition that all nodes adopt the same opinion is mainly analyzed. In addition, the opinion evolution in this paper is based on the bounded confidence model.

Although the models of opinion formation are various, the previous studies always focus on how opinions evolve to the consensus or the polarization states [13,18–20,26,27]. For instance, Lorenz [19] indicates that polarization state may occur if individuals just assimilate the opinions to which they agree. On the contrary, the assimilation of all opinions usually leads to consensus state [13]. On the other hand, some studies analyze the co-evolution of opinion and network structure [26,27]. Additionally, Iyengar, Van den Bulte, and Valente [17] discussed the fundamental role of opinion leader in the diffusion of new product. However, these studies focus on the properties of the opinion evolution rather than a feasible method to control the opinion formation.

Some studies also care the feasible methods to control the opinion formation [28,29]. Afshar and Asadpour [28] discuss the influence of informed agents in opinion evolution process. The informed agent in their study is an agent that gradually and intentionally changes its neighbors’ opinions towards the desired direction. Besides, the major agents are not informed agents and do not know who is informed agent. Based on various simulations, they analyze how the number of informed agents, the nodal degree and network structure influence the opinion evolution. AskariSichani and Jalili [29] further discuss how to maximize the influence of informed agents by connecting them to suitable node. In detail, the informed agents should be connected to the nodes with small in-degrees and high out-degree that are connected to high in-degree nodes. Then, the public opinion will be significantly changed towards the desired direction. The main advantage of this method is that the public opinion is very close to desired opinion after the evolution process. However, this method is based on a long process in which nodes repeatedly interact with their neighbors. If the process lasts a lot of time, there may be some noisy factors in this process, such as the change of network structure [3]. Therefore, we present a method based on a diffusion process that lasts a short time so that many noisy factors can be ignored.

2.3. The issue of controlling diffusion process

Controlling the diffusion process is an important motivation of diffusion analyses. Influence maximization problem is the main aspect of controlling the diffusion process [7,14,30–33]. Kempe, Kleinberg and Tardos [7] raise the issue of influence maximization through a social network. The problem is to find a set of nodes to be active initially such that the expected number of activated nodes after the diffusion terminates is maximized. Chen et al. [14] discuss the diffusion process in which opposite information may emerge and propagate, and they present algorithm to maximize the spread of the positive influence. Chen, Lu and Zhang [30] present a study that care the influence maximization problem in a time-critical situation. Their algorithm outperforms previous algorithms that disregard the deadline constraint and delays in diffusion. Besides, Feng et al. [31] argue that past studies have not considered the impact of novelty decay on influence propagation. In their study, repeated exposures will have diminishing influence on users. However, in this paper, the traditional influence maximization algorithms are proved inapplicable to the control of the uncertain opinion diffusion.

3. Model statements

In this section, we describe our model by combining the independent cascade model [3,7] and the evolution of opinions.

3.1. Definition of individual opinion

Let $O(i, t) \in [-1, 1]$ denote node $i$'s opinion at time $t$ [18,19]. Here, $O(i, t) = -1$ and $O(i, t) = 1$ denote opposite extreme opinions. Moreover, let $O_0 \in [-1, 1]$ denote the opinion that is contained in the information. In other words, $O_0$ is the opinion of the initiator who triggers the diffusion process. In this paper, $O_0$ cannot be affected by others' opinions, and it is the initiator's opinion at any time.
3.2. Diffusion model with individual opinion

Let $G = (N, E)$ be a social network and every node in $G$ is considered as an individual. Let $X(i, t)$ denote the state of node $i$ at discrete time $t$. Then, we have

$$X(i, t) = \begin{cases} 0, & \text{inactive;} \\ 1, & \text{active.} \end{cases}$$

(1)

Here, $X(i, t) = 0$ means that node $i$ has not received the information or does not perform dissemination behavior after receiving the information. Correspondingly, $X(i, t) = 1$ means that node $i$ has diffused the information.

According to the independent cascade model, node can try to activate its neighbors only once after it becomes active. Besides, in this paper, each active node diffuses not only the information, but also its opinion on the information.

Then, we define the influence of opinions on diffusion probability. Node $i$ activates node $j$ with the probability:

$$p_{ij} = \begin{cases} p_0 e^{-|O(i, t) - O(j, t)|}, & \text{node } i \text{ is the initiator;} \\ p_0 e^{-\frac{1}{2} |O(i, t) - O(j, t)| - \frac{1}{2} |O_0 - O(i, t)|}, & \text{otherwise.} \end{cases}$$

(2)

Here, $p_0$ is a basic probability, and $p_{ij}$ is the modifications of $p_0$ [34].

Each node may change its opinion if it is exposed to the information. Let $A_i$ denote the set of opinions which can influence $O(i, t)$, and $A_i$ is $\phi$ at the beginning of each time step. If node $i$’s neighbor node $j$ is active and $|O(i, t) - O(j, t)| < \varepsilon_i$, $O(j, t)$ will be added to $A_i$. Besides, if $|O_0 - O(i, t)| < \varepsilon_i$, $O_0$ will also be added to $A_i$. Then, we have

$$O(i, t + 1) = \frac{O(i, t) + \sum_{O(j, t) \in A_i} O(j, t)}{1 + |A_i|}.$$  

(3)

Here, Eq. (3) is based on the bounded confidence model, and $\varepsilon_i$ is a confidence bound [19]. Besides, each opinion can only influence $O(i, t)$ no more than once. In other words, although a node may receive the information several times, the opinion $O_0$ can influence the node only once. Moreover, if node $j$’s opinion changes after it influences $O(i, t)$, it will not influence $O(i, t)$ again.

In addition, to help clearly describe the proposed theoretical analysis framework, Table 1 provides the main symbols used in this paper and their specific meanings.

As shown in Fig. 1, each node except the initiator has four possible states. State 1 is the initial state. The possible state transitions are:

- A node in State 1 is activated at this time step. Its active neighbors’ opinions and $O_0$ are not in its confidence interval. The node will be in State 3 at next time step.
- A node in State 1 is not activated at this time step. Its active neighbors’ opinions or $O_0$ are in its confidence interval. The node will be in State 2 at next time step.
- A node in State 1 is activated at this time step. Its active neighbors’ opinions or $O_0$ are in its confidence interval. The node will be in State 4 at next time step.
- A node in State 2 receives the information again and is activated at this time step. The node will be in State 4 at next time step.
- A node in State 3 receives the information again, and its active neighbors’ opinions or $O_0$ are in its confidence interval. The node will be in State 4 at next time step.

4. Properties of the diffusion model

In this section, we briefly discuss the properties of the diffusion model based on theoretical analyses and simulations. The simulations are in two empirical networks (Enron Email network [35,36] and Facebook network [37]). In the Email network, there are 36 692 nodes while the mean degree is 10.26. Besides, there are 63 731 nodes in the Facebook network, and the mean degree is 25.3.
Fig. 1. The state transition of each node is shown in this figure. The green state means activation. $O(i, t) \neq O(i, 0)$ means that the node’s opinion has been influenced by others’ opinions. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 2. The illustration of the three probability density functions: (i) $f_1(x)$; (ii) $f_2(x)$; (iii) $f_3(x)$.

In the simulations, the basic probability $p_0 = 0.1 \times e = 0.2718$ so that $p_{i,j} \in [0.0368, 0.2718]$. Based on previous study [38], it can be known that the thresholds that limit the spread of information in the two empirical networks are between 0.0368 and 0.2718. Therefore, when $p_0 = 0.2718$, the initiator’s opinion can usually widely spread but not always activate the most nodes. Then, the simulated phenomena will be more obvious. We will not test different basic probability because various initial distributions of opinions and confidence bounds can cause different diffusion probabilities. Each trial is performed with 1000 replications. The node with the largest degree will be set as the unique initiator. For the sake of simplification, it is assumed that all the nodes have the same confidence bound $\varepsilon_i$.

Three probability density functions [39] are used to represent the typical states [18,19] of the initial opinions. In simulations, the initial opinion of each node will be generated randomly based on the probability density functions. The three probability density functions are shown in the following and Fig. 2:

1. $f_1(x) = 0.5$, $x \in [-1, 1]$. This state indicates the absence of convergent opinions in social networks. For instance, individuals hold scattered opinions on some fuzzy events because nobody knows the truth.

2. $f_2(x) = \frac{5}{2\pi} e^{-12.5x^2}$, $x \in [-1, 1]$. This state represents that most social individuals hold neutral opinion. For instance, most people hold neutral opinions on some insipid topic, and they have no obvious preference.

3. $f_3(x) = \begin{cases} \frac{5}{2\pi} e^{-12.5(x+1)^2}, & x < 0; \\ \frac{5}{2\pi} e^{-12.5(x-1)^2}, & x \geq 0. \end{cases}$, $x \in [-1, 1]$. Opposite to $f_2(x)$, this probability density function indicates the polarized opinions which means nodes in social networks are generally distributed in two camps. For instance, individuals hold polarized opinions on some controversial issues.
Then, we discuss which value of $O_0$ will lead to the largest active rate.

**Theorem 1.** Let $g(O_0) = r_0$. Besides, let $\mu$ denote the mathematical expectation of $f_0(x)$. It is assumed that there exist $\delta \in [-1, 1]$ and $\omega \geq 0$ satisfy the condition: $\int_{-1}^1 e^{-x+y}f_0(x)dx = \int_{y}^1 e^{-x+y}f_0(x)dx > 0$ when $y \in [\delta - \omega, \delta]$ and $\int_{-1}^1 e^{-x+y}f_0(x)dx - \int_{-1}^y e^{-x+y}f_0(x)dx < 0$ when $y \in (\delta, \delta + \omega)$. If $\epsilon_1 > 0$ and $\mu \in [\delta - \omega, \delta + \omega]$, there exists a value $\xi \in [\min(\mu, \delta), \max(\mu, \delta)]$ so that $g(\xi)$ is the maximum of $g(O_0)$.

**Proof.** Let

$$f(y) = \int_{-1}^1 p_0 e^{-|x-y|}f_0(x)dx.$$ 

Therefore,

$$f(y) = \int_{-1}^y p_0 e^{-x-y}f_0(x)dx + \int_y^1 p_0 e^{-x+y}f_0(x)dx$$

$$= p_0 \left[ e^{-y} \int_{-1}^y e^{y}f_0(x)dx + e^{y} \int_y^1 e^{-y}f_0(x)dx \right].$$

For $\forall y_0 \in [-1, 1]$,

$$\lim_{\Delta y \to 0} \frac{f(y_0 + \Delta y) - f(y_0)}{\Delta y} = \lim_{\Delta y \to 0} \left\{ \frac{p_0 \left[ e^{-y_0+\Delta y} \int_{-1}^{y_0+\Delta y} e^{y_0+\Delta y} f_0(x)dx + e^{y_0+\Delta y} \int_{y_0+\Delta y}^1 e^{-y_0+\Delta y} f_0(x)dx \right]}{\Delta y} - \frac{p_0 \left[ e^{-y_0} \int_{-1}^{y_0} e^{y_0} f_0(x)dx + e^{y_0} \int_{y_0}^1 e^{-y_0} f_0(x)dx \right]}{\Delta y} \right\}$$

$$= \lim_{\Delta y \to 0} \left\{ \frac{p_0 e^{-y_0} \left[ (e^{\Delta y} - 1) \int_{-1}^{y_0} e^{y_0} f_0(x)dx + e^{-\Delta y} \int_{y_0}^{y_0+\Delta y} e^{y_0} f_0(x)dx \right]}{\Delta y} - \frac{p_0 e^{y_0} \left[ (e^{\Delta y} - 1) \int_{y_0}^{y_0+\Delta y} e^{-y_0} f_0(x)dx - e^{\Delta y} \int_{y_0}^{y_0+\Delta y} e^{y_0} f_0(x)dx \right]}{\Delta y} \right\}$$

Let $z = e^{-\Delta y} - 1$ so that $\Delta y = -\ln(z + 1)$. Therefore,

$$\lim_{\Delta y \to 0} \frac{e^{-\Delta y} - 1}{\Delta y} = \lim_{\Delta y \to 0} \frac{\ln(z + 1)}{\Delta y} = \lim_{z \to 0} \frac{z}{\ln(z + 1)} = e.$$ 

It is easy to know that $\ln(z + 1)^{\frac{1}{z}} = e$. It means that $\lim_{\Delta y \to 0} \frac{e^{-\Delta y} - 1}{\Delta y} = -1$. Correspondingly, $\lim_{\Delta y \to 0} \frac{e^{\Delta y} - 1}{\Delta y} = 1$. Based on the definition of integral, it can be known that $\int_{y_0}^{y_0+\Delta y} e^{y_0} f_0(x)dx = \lim_{n \to \infty} \frac{\Delta y}{n} \sum_{i=1}^n e^{y_0} f_0(\eta_i)$ while $y_0 = \eta_0 < \eta_1 < \eta_2 < \cdots < \eta_n = y_0 + \Delta y$. Then,

$$\lim_{\Delta y \to 0} \frac{e^{-\Delta y} \int_{y_0}^{y_0+\Delta y} e^{y_0} f_0(x)dx}{\Delta y} = \lim_{n \to \infty} \frac{\Delta y}{n} \sum_{i=1}^n e^{y_0} f_0(\eta_i)$$

$$= \lim_{\Delta y \to 0} \left[ e^{-\Delta y} \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^n e^{y_0} f_0(\eta_i) \right].$$

Here, $\Delta y \to 0$ means $\eta_i \to y_0$. Therefore, $\lim_{\Delta y \to 0} \frac{e^{-\Delta y} \int_{y_0}^{y_0+\Delta y} e^{y_0} f_0(x)dx}{\Delta y} = e^{y_0} f_0(y_0)$. Correspondingly, $\lim_{\Delta y \to 0} \frac{e^{\Delta y} \int_{y_0}^{y_0+\Delta y} e^{y_0} f_0(x)dx}{\Delta y} = e^{y_0} f_0(y_0).$
Therefore, \( \lim_{\Delta y \to 0} \frac{f(y_0 + \Delta y) - f(y_0)}{\Delta y} \) exists for all \( y_0 \in [-1, 1] \) so that \( f(y) \) satisfies the derivable condition on \([-1, 1]\). Then,

\[
 f'(y) = p_0 \int_{-1}^{1} e^{-x|y|} f_0(x) \, dx - \int_{-1}^{y} e^{-y|y|} f_0(x) \, dx.
\]

Based on the condition in Theorem 1, it can be known that \( f'(y) > 0 \) when \( y \in [\delta - \omega, \delta] \) and \( f'(y) < 0 \) when \( y \in (\delta, \delta + \omega) \). Therefore, \( f(\delta) \) is the local maximum of \( f(y) \).

It can be obtained that

\[
 \bar{p}_{t=1} = \int_{-1}^{1} p_0 e^{-|x| - |y|} f_0(x) \, dx.
\]

Therefore, \( \bar{p}_{t=1} = f(O_0) \). It means that \( \bar{p}_{t=1} \) is maximum at \( O_0 = \delta \).

Besides,

\[
 \bar{p}_{t=1} = \int_{-1}^{1} f_1(y) \left[ \int_{-1}^{1} p_0 e^{-\frac{1}{2}|x| - \frac{1}{2}|y| - \frac{1}{2}|y| - |y|} f_0(x) \, dx \right] \, dy.
\]

If \( \varepsilon_i > 0 \), nodes’ opinions may be influenced by \( O_0 \) and their neighbors’ opinions. The co-influence of a node’s neighbors’ opinions leads the node’s opinion to change towards the mathematical expectation of its neighbors’ opinions. Therefore, nodes’ opinions change towards \( O_0 \) and \( \mu \). It means that \( f_i(y) \) focuses on a value between \( O_0 \) and \( \mu \). If \( O_0 \) approaches \( \mu \), the co-influence will be larger so that \( f_i(y) \) will be more focused. Then, \( \bar{p}_{t=1} \) will also be larger. Therefore, along with the opinion evolution, the value of \( O_0 \) that leads to the largest \( \bar{p}_{t=1} \) will change from \( \delta \) to \( \mu \).

Overall, there exists a value \( \zeta \in [\min(\mu, \delta), \max(\mu, \delta)] \) so that \( g(\zeta) \) is the maximum of \( g(O_0) \). □

Theorem 1 presents the possible interval of \( O_0 \) which will lead to the largest active rate. Then, we discuss the simulated data in Fig. 3. Considering the value \( \delta \) in Theorem 1, based on Matlab, we obtain that if \( f_0(x) = f_i(x) \) or \( f_0(x), f'_i(y) > 0 \) when \( y \in [-1, 0] \) and \( f'(y) < 0 \) when \( y \in (0, 1] \). It means that \( \delta \) exists and \( \delta = 0 \). If \( f_0(x) = f_{\text{def}}(x), f'_i(y) > 0 \) when \( y(-1, -0.869) \cup (0.869, 1) \) and \( f'(y) < 0 \) when \( y(-0.869, 0) \cup (0.869, 1) \). Therefore, \( \delta \approx 0.869 \) or \(-0.869\). Besides, the mathematical expectations of the three probability density functions are zero. Therefore, based on Theorem 1, it can be known that if \( f_0(x) = f_i(x) \) or \( f_{\text{def}}(x) \), the active rate reaches maximum when \( O_0 = 0 \). If \( f_0(x) = f_{\text{def}}(x) \), the maximum of active rate are in the intervals \( O_0 \in [-0.869, 0.869] \). These phenomena can be observed in Fig. 3.

There is an interesting phenomenon about the fluctuation of the active rate. In Fig. 3.1(a) (b) and 3.2(a) (b), the fluctuation of the active rate is not acute when \( \varepsilon_i = 1 \). However, in Fig. 3.1(c) and 3.2(c), the fluctuation of the active rate is quite acute when \( \varepsilon_i = 1 \). The reason is that large \( \varepsilon_i \) makes nodes’ opinions focus on \( O_0 \) when \( f_0(x) \) is not polarized. Therefore, the diffusion probability is usually large when \( \varepsilon_i = 1 \). Then, the fluctuation of the active rate is not acute. On the contrary, if \( f_0(x) \) is polarized \( (f_0(x) = f_{\text{def}}(x)) \), \( \varepsilon_i = 0.5 \) cannot make \( O_0 \) influence the two poles simultaneously so that the diffusion probability is quite small. However, when \( \varepsilon_i = 0 \) or \( O_0 = 0 \) can influence the two poles simultaneously while \( O_0 = -1 \) or \( 1 \) cannot do. Therefore, in Fig. 3.1(c) and 3.2(c), the fluctuation of the active rate is more acute when \( \varepsilon_i = 1 \).

In addition, the fluctuation of the active rate is acute if \( f_0(x) = f_i(x) \). It means that \( O_0 \) can significantly influence the active rate if most nodes hold similar opinions initially. When nodes’ initial opinions are polarized, the influence of \( O_0 \) will be limited. Moreover, in Fig. 3.1(a) (c) and 3.2(a) (c), larger \( \varepsilon_i \) leads to larger active rate. The reason is that large \( \varepsilon_i \) makes each node’s opinion be able to influence most other nodes. Then, the opinions will evolve to the consensus state because of the interaction. Similar conclusion has been presented in Refs. [13,19]. With the convergence of opinions, the average diffusion probability will increase. On the contrary, if \( \varepsilon_i \) is small, the opinions will not necessarily be convergent. The final distribution of opinions heavily depends on the initial opinions. For instance, if \( f_0(x) = f_i(x) \), the opinions may become several opinion clusters, and then these opinion clusters cannot influence other clusters. Therefore, the diffusion probability is usually small if \( \varepsilon_i \) is small. The exception occurs in Fig. 3.1(b) and 3.2(b), \( r_0 \) with \( \varepsilon_i = 0.5 \) may be larger than \( r_0 \) with \( \varepsilon_i = 1 \) when \( O_0 \) is near 0.75 or −0.75. It is because that the opinions are convergent initially, and then opinions can be considered as an opinion cluster. Larger \( \varepsilon_i \) will not make the opinions be more convergent. In this case, \( O_0 = 0.75 \) or −0.75 can influence the opinion cluster when \( \varepsilon_i = 1 \) but it cannot when \( \varepsilon_i = 0.5 \). The influence of \( O_0 = 0.75 \) makes the opinion cluster become more scattered in Fig. 3.1(b) and 3.2(b). Therefore, the active rate with \( \varepsilon_i = 0.5 \) is larger in this special case. To sum up, larger \( \varepsilon_i \) leads to larger active rate in major cases. In real world, it means that the opinion can spread widely among the people who will assimilate the opinion that is quite different from their opinions.

Empirically, the knowledge on an objective will influence people’s opinions and the spread of a certain opinion. In detail, when people have little knowledge or data about an objective, they do not have clear opinions. Because of conformist mentality, many people just join the mainstream faction and express the opinion that is the most popular. Therefore, the diffusion probability is usually large so that a rumor will spread fast, just like when \( f_0(x) = f_i(x) \). On the contrary, when people have enough knowledge or data, they will hold clear opinions. The opinion divergence may be more significant in this case. Therefore, a certain opinion will spread slowly.
5. The algorithm to control the uncertain opinion diffusion process

In this section, we discuss how to control the uncertain opinion diffusion process. Nowadays, online social networks have been greatly popular through which millions of users are connected. It has been known that the interactions in online social networks can change people’s opinions and their choices [10,13]. For instance, the information in Twitter has proved to be very powerful in the 2008 US presidential elections [40,41]. Therefore, we present an optimization problem at the background of voting. It is assumed that each node can choose one of the two candidates or abstention based on its opinion. As shown in the model statements, the nodes’ opinions may change in the diffusion process. Therefore, the voting result will be influenced. Then, we formulate the problem of which the goal is to set suitable initiators and their opinions to maximize the advantage of one candidate.

5.1. Voting mechanism

When a node is asked to vote on an issue, it has three choices. Let $V_i$ denote the choice of node $i$:

$$V_i = \begin{cases} -1; \\ 0; \\ 1. \end{cases}$$

(4)

Here, $V_i = -1$ can represent the support to one of the two candidates while $V_i = 1$ represents the opposite choice. Meanwhile, $V_i = 0$ represents abstention.

Each node makes a choice based on its opinion. Then, we define the mapping from opinion to the probabilities of making choices:

$$p(V_i = -1) = \begin{cases} -O(i, t), & O(i, t) < 0; \\ 0, & \text{otherwise}. \end{cases}$$

(5)
\[ p(V_i = 1) = \begin{cases} \mathbb{O}(i, t), & O(i, t) > 0; \\ 0, & \text{otherwise}. \end{cases} \] (6)

\[ p(V_i = 0) = \begin{cases} 1 - p(V_i = 1), & O(i, t) > 0; \\ 1 - p(V_i = -1), & O(i, t) < 0; \\ 1, & O(i, t) = 0. \end{cases} \] (7)

In this section, we will develop strategy to maximize the advantage of one candidate at the end of diffusion. Therefore, \( O(i, t) \) in the three equations is node \( i \)'s opinion at the end of diffusion by default.

5.2. Problem formulation and properties

Then, we formulate an optimization problem, called Voting Victory Problem (VVP). The voting victory problem is formally defined as follows.

**Voting Victory Problem (VVP).** Let two sets of nodes \( L = \{i | V_i = -1\} \) and \( R = \{i | V_i = 1\} \). Given \( k > 0 \), the goal is to select a set \( S \) of \( k \) initiators and set their opinion \( O_0 \) so that \( |R| - |L| \) is maximized at the end of diffusion.

The confidence bound \( \epsilon_i \) and the initial distributions of opinions are uncontrollable in VVP. We can just develop strategy based on the estimations of them. Therefore, \( \epsilon_i \) and the initial distributions of opinions are assumed to be known in the problem. For the sake of simplification, it is assumed that all the nodes have the same confidence bound \( \epsilon_i \).

Overall, we have two sub-tasks for VVP:

- Set the parameter \( O_0 \);
- Select \( k \) initiators to send information.

The selection of initiators will be influenced by \( O_0 \). Therefore, we will estimate \( O_0 \) first. **Theorem 2** shows the property of the problem.

**Theorem 2.** VVP is \( NP \)-hard.

**Proof.** It is assumed that the clustering coefficient of the network is small, and the initial distribution of opinion is scattered. If \( \epsilon_i \) is small enough, for each node \( i \) and its neighbor node \( j \), the inequality \( |O(i, t) - O(j, t)| > \epsilon_i \) can be satisfied. It means that any nodes cannot influence their neighbors' opinions, and only \( O_0 \) is able to influence nodes' opinions. In this case, VVP is equivalent to traditional influence maximization problem because larger scope of diffusion will make \( O_0 \) influence more nodes. In other words, traditional influence maximization problem can be considered as a special case of VVP. Traditional influence maximization problem has been proved as \( NP \)-hard \[7\]. Therefore, VVP is also \( NP \)-hard. \( \square \)

5.3. Theoretical analyses and algorithm design

5.3.1. The estimation of the most suitable \( O_0 \)

Let \( O_d \) denotes the most suitable opinion for initiator in VVP. In other words, \( |R| - |L| \) will be maximized if \( O_0 = O_d \). Then, we estimate the value of \( O_d \).

The confidence bound \( \epsilon_i \) decides the interval of opinions that \( O_0 \) can influence directly. It means the actual interval that \( O_0 \) can influence is \( [O_0 - \epsilon_i, O_0 + \epsilon_i] \). Besides, when a node become active, it diffuses not only the information that contains \( O_0 \), but also its own opinion. \( O_0 \) may have influence on the active node's opinion. Therefore, active node's opinion should also be considered. Then, we estimate the average opinion that is expressed by the active nodes. Let

\[ w_1 = \int_{O_0 - \epsilon_i}^{O_0} f_0(x) dx \] (8)

and

\[ w_2 = \int_{O_0}^{O_0 + \epsilon_i} f_0(x) dx. \] (9)

Here, \( w_1 + w_2 \) denotes the rate of the opinions that can be influenced by \( O_0 \). Then, we have

\[ l_1 = \frac{1}{w_1} \int_{O_0 - \epsilon_i}^{O_0} x f_0(x) dx \] (10)
and
\[
l_2 = \frac{1}{w_2} \int_{O_0}^{O_0 + \varepsilon_1} x f_0(x) dx. \tag{11}
\]

Here, \( l_1 \) denotes the mathematical expectations of opinions that are in the confidence interval of \( O_0 \) and smaller than \( O_0 \). \( l_2 \) denotes the mathematical expectations of opinions that are in the confidence interval of \( O_0 \) and larger than \( O_0 \). Based on \( l_1 \) and \( l_2 \), we can estimate the active nodes’ opinions which have been influenced by \( O_0 \).

Here, \( l'_1 = \frac{1}{2}(O_0 + l_1) \) \( (12) \)

and
\[
l'_2 = \frac{1}{2}(O_0 + l_2). \tag{13}
\]

Here, \( l'_1 \) denotes average opinion that is expressed by the active nodes whose initial opinions are smaller than \( O_0 \). Correspondingly, \( l'_2 \) denotes average opinion that is expressed by the active nodes whose initial opinions are larger than \( O_0 \). Based on \( O_0, l'_1 \) and \( l'_2 \), we can estimate the co-influence of them. Therefore, we have
\[
O' = \frac{O_0 + w_1 l'_1 + w_2 l'_2}{1 + w_1 + w_2}. \tag{14}
\]

Here, \( O' \) represents the composite opinion of \( O_0, l'_1 \) and \( l'_2 \). The opinions in \([O_0 - \varepsilon_1, O_0] \) change towards positive direction so that the \( p(V_i = 1) \) may increase and \( p(V_i = -1) \) may decrease. It means that \(|R| - |L|\) will increase. However, the opinions in \([O_0, O_0 + \varepsilon_1] \) change towards negative direction so that \(|R| - |L|\) decreases. Therefore, the initiator should try to increase \( \int_{O_0 - \varepsilon_1}^{O_0} f_0(x) dx \) and decrease \( \int_{O_0}^{O_0 + \varepsilon_1} f_0(x) dx \). Besides, for the opinions in \([O_0 - \varepsilon_1, O_0 + \varepsilon_1] \), the degrees of the influence of \( O_0 \) are not the same. Therefore, \(|O_0 - x| \) should also be considered. Then, we have
\[
\rho_1 = \int_{O_0 - \varepsilon_1}^{O_0} (O' - x) f_0(x) dx - \int_{O_0}^{O_0 + \varepsilon_1} (O' - x) f_0(x) dx. \tag{15}
\]

Here, \( \rho_1 \) denotes the influence of \( O' \). Moreover, \( l'_1 \) and \( l'_2 \) may influence the opinions that are not in the confidence interval of \( O_0 \). Therefore, it can be obtained that
\[
\rho_2 = \int_{l'_1 - \varepsilon_1}^{l'_1} (l'_1 - x) f_0(x) dx \tag{16}
\]

and
\[
\rho_3 = \int_{l'_2 - \varepsilon_1}^{l'_2 + \varepsilon_1} (l'_2 - x) f_0(x) dx. \tag{17}
\]

Overall, we can obtain that
\[
\sigma(O_0) = \rho_1 + w_1 \rho_2 + w_2 \rho_3. \tag{18}
\]

Then, \( \sigma(O_0) \) should be the maximum of \( \sigma(O_0) \).

5.3.2. The selection of the initiators

In this section, we present algorithm to select the initiators.

Traditional influence maximization algorithms usually select the node that has the most neighbors. When the number of initiators is more than one, the set of nodes that have the most neighbors totally will be selected. For instance, Greedy algorithm \([30] \) is a classic algorithm in influence maximization problem. In Greedy algorithm, the node that maximizes the increment of neighbors will be selected as a new initiator. Let \( n_i \) denote the number of neighbors of node \( i \). Besides, let \( F(i) = n_i \) and \( F(S) \) be the number of the total neighbors of the nodes in set \( S \). Then, the Greedy algorithm is shown in Algorithm 1.

**Algorithm 1. Greedy algorithm**

1) initialize \( S = \phi \)
2) for \( i = 1 \) to \( k \) do
3) \( \) select \( u = \arg \max_{w \in N \setminus S} (F(S \cup \{w\}) - F(S)) \)
4) \( \) \( S = S \cup \{u\} \)
5) end for
6) output \( S \)
The performance of Greedy algorithm is quite good. However, the runtime of Greedy algorithm is very high when the number of initiators or the number of nodes increases. In order to reduce the runtime of algorithm, Chen, Wang and Yang present the DegreeDiscount algorithm [42]. The runtime of DegreeDiscount algorithm is much lower than that of Greedy algorithm. The DegreeDiscount algorithm is shown in Algorithm 2.

**Algorithm 2. DegreeDiscount algorithm**

1) initialize $S = \phi$
2) for $i = 1$ to $|N|$ do
3) \[ dd_i = F(i) \]
4) initialize $t_i$ to 0
5) end for
6) for $i = 1$ to $k$ do
7) select $u = \arg\max_w \{dd_w | w \in N \setminus S\}$
8) $S = S \cup \{u\}$
9) for each neighbor $v$ of $u$ and $v \in N \setminus S$ do
10) \[ t_v = t_v + 1 \]
11) \[ dd_v = d_v - 2t_v - (d_v - t_v)t_vp \]
12) end for
13) end for
14) output $S$

In DegreeDiscount algorithm, $p$ is the diffusion probability. However, the diffusion probability in our model is uncertain. Therefore, in this section, $p$ is replaced by the average probability $\bar{p}_i$.

The goal of the two traditional algorithms is to select a set of nodes to maximize the active rate $r_a$. However, the maximum $r_a$ is not the request in VVP. If $r_a$ is large, nodes may receive the information more than once. As mentioned in the model statements, each node can be influenced by $O_0$ at most once. Therefore, larger $r_a$ cannot increase the influence of $O_0$. On the contrary, a node is influenced by various opinions when it receives the information more than once. Then, the influence of $O_0$ will be diluted. On the other hand, larger $r_c$ can make $|R| - |L|$ become larger if $O_0$ is suitable. The reason is that larger $r_c$ means more nodes can be exposed to the information and may be influenced by $O_0$. Therefore, a larger $r_c$ with a smaller $r_a$ is needed in VVP. In order to cause large $r_c$ with small $r_a$, we present a heuristic algorithm to select the initiators. Then, we will compare the heuristic algorithm with the Greedy algorithm and DegreeDiscount algorithm in the simulations.

In order to decrease $r_a$, the degrees of initiators should not be very large. However, a too small degree will lead to small $r_c$. Therefore, if node $i$ is chosen as initiator, $n_i$ should satisfy the inequality

$$n_i \ast \bar{p}_i > \frac{1}{\bar{p}_i}. \quad (19)$$

When $k$ increases, the limitation of the degrees of initiators can be weakened. Then, the inequality becomes

$$n_i \ast \bar{p}_i \ast k > \frac{1}{\bar{p}_i}. \quad (20)$$

If the inequality above is satisfied, $r_c$ will be an acceptable value. Besides, the interval of opinions that $O_0$ can influence should also be considered. Therefore,

$$n_i \geq \frac{1}{\int_{0_0 - \epsilon}^{0_0} f_0(x) dx}. \quad (21)$$

In the heuristic algorithm, the nodes that cannot satisfy the inequality (20) and (21) will be excluded. Besides, in some unconnected networks, a node with small degree may be isolated. In order to make the information be able to spread from the initiator to most nodes, the isolated nodes will also be excluded. Then, among the nodes that are not excluded, the $k$ nodes with the smallest degrees will be selected. In addition, if the number of nodes that are not excluded is less than $k$, the $k$ nodes with the largest degrees in the network will be selected. Overall, Algorithm 3 is obtained.

Then, we discuss the time complexity of Algorithm 3. As mentioned above, the nodes with small degree will be selected in Algorithm 3. Therefore, the probability that the initiators are the neighbor of each other is very small. It means that the connection between initiators can be ignored. It is noteworthy that considering the connection between the initiators leads to the large complexity of Greedy algorithm. Therefore, the complexity of Algorithm 3 is much smaller than that of Greedy algorithm. If the number of nodes is $|N|$ and the number of initiators is $k$, the time complexity of Algorithm 3 will be $O(|N| \ast \sqrt{|N|} + k \ast |N|)$. 


5.4. Experiments

In this section, we test our theoretical estimation and algorithm by simulations. The basic probability $p_0 = 0.1 \times e = 0.2718$. Each trial is performed with 1000 replications.

5.4.1. The most suitable $O_0$

In each trial, $k$ initiators will be selected randomly, and $k = 1, 11, \ldots, 101$. In the simulations, the initiator’s opinion $O_0 = \{-1, -0.9, -0.8, \ldots, 0.8, 0.9, 1\}$, and the value which can cause the largest simulated $|R| - |L|$ is the most suitable $O_0$.

As shown in Tables 2 and 3, the most suitable $O_0$ in Email network and Facebook network are the same. Besides, in the two networks, the theoretical values are equal to the simulated values in most cases. The mistake just occurs when $f_{0}(x) = f_{\overline{0}}(x)$ and $\epsilon_i = 1$, and the distance between the theoretical value and the simulated value is 0.1. In Tables 2 and 3, the mean distance between the theoretical values and the simulated values is 0.0167. In addition, the most suitable $O_0$ is constant when $k$ is varied. The reason is that the fluctuation of $r_a$ and $r_i$ is not acute enough to change the most suitable $O_0$.

5.4.2. The initiators

In this section, the performance of the heuristic algorithm is tested. The initiators in various simulations are selected based on the three algorithms or selected randomly. Besides, $O_0$ is set as the most suitable value obtained in above simulations.

Figs. 4 and 5 show the performance of different algorithms. The curves are the simulated values of $(|R| - |L|)/|N|$ with different initiators. Besides, $\epsilon_i = 0.5$ in Fig. 4 while $\epsilon_i = 1$ in Fig. 5. It can be known from the figures that Heuristic algorithm can lead to expected results. When $k$ is small, none of the four curves is always above others. However, when $k$ increases, Heuristic algorithm usually leads to the largest $|R| - |L|$, especially when $k > 50$.

As shown in Figs. 4 and 5, the initial distribution of opinions and the value of $\epsilon_i$ can influence the performances of algorithms. In Fig. 4, if $f_0(x) = f_\overline{0}(x)$, the simulated data based on different algorithms are approximately overlapping, and the distances between them are quite small. When $f_\overline{0}(x) = f_i(x)$ or $f_{\overline{i}}(x)$, the advantage of the Heuristic algorithm is more obvious. The reason is that opinions are scattered when $f_\overline{0}(x) = f_i(x)$ and $\epsilon_i$ is small in Fig. 4. The fluctuation of active rate

---

**Algorithm 3.** Heuristic algorithm for VVP

1. initialize $S = \phi$
2. initialize $V = \phi$
3. initialize $w = 0$
4. for $i = 1$ to $|N|$ do
   5. initialize $b_i = 0$
   6. end for
7. select $v = \text{argmax}_{w \in N}(F(w))$
8. $b_v = 1$
9. for each edge $(i, j) \in \{(i, j) | (i, j) \in E, b_i = 0, b_j = 1\}$ do
   10. $b_i = 1$
   11. end for
12. for $i = 1$ to $|N|$ do
   13. if $b_i = 1$ and $F(i) \ast \overline{b}_i \ast k \geq \frac{1}{p_0}$ and $F(i) \geq \frac{1}{|f_{0}(x)| \text{dx}}$ do
      14. $V = V \cup \{i\}$
   15. end if
   16. end for
17. if $|V| \geq k$ do
   18. for $i = 1$ to $k$ do
      19. select $u = \text{argmin}_{w \in (V \setminus S)}(F(w))$
   20. $S = S \cup \{u\}$
   21. end for
   22. end if
   23. else do
      24. for $i = 1$ to $k$ do
         25. select $u = \text{argmax}_{w \in (N \setminus S)}(F(w))$
      26. $S = S \cup \{u\}$
      27. end for
   28. end else
   29. output $S$

---
Table 2
Initiator’s most suitable opinion in Email network.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$f_i(x)$</th>
<th>$\epsilon_i$</th>
<th>$O_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>Theoretical 1</td>
<td>Simulated 1</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>Theoretical 0.4</td>
<td>Simulated 0.4</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>Theoretical $-0.5$</td>
<td>Simulated $-0.5$</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>Theoretical $-0.3$</td>
<td>Simulated $-0.4$</td>
<td></td>
</tr>
</tbody>
</table>

Table 3
Initiator’s most suitable opinion in Facebook network.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$f_i(x)$</th>
<th>$\epsilon_i$</th>
<th>$O_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>Theoretical 1</td>
<td>Simulated 1</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>Theoretical 0.4</td>
<td>Simulated 0.4</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>Theoretical $-0.5$</td>
<td>Simulated $-0.5$</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>Theoretical $-0.3$</td>
<td>Simulated $-0.4$</td>
<td></td>
</tr>
</tbody>
</table>

does not influence the $|R| - |L|/|N|$ obviously in this case. In Fig. 5, if $f_0(x) = f_i(x)$, the heuristic algorithm is significantly better than other algorithms. The reason is that $\epsilon_i$ is large in Fig. 5 so that larger active rate increases the distance between the performances of algorithms. In addition, if the initial distribution of opinions, network structure and algorithm are fixed, the performance of algorithm in Fig. 5 is always better than that in Fig. 4. The reason is that larger $\epsilon_i$ makes opinions become convergent, and $O_0$ is able to influence more opinions.

In addition, it is unexpected that $|R| - |L|$ based on Greedy algorithm and DegreeDiscount algorithm are smaller than that based on random selection in many cases. It means that traditional influence maximization algorithms are not applicable to VVP. In other words, influencing people’s opinions and their choices in voting may be significantly different from traditional influence maximization problem. The reason is that the goal of the traditional influence maximization algorithms is to maximize the number of people who are influenced, not to maximize the total influence on the people. However, in this problem, the essential goal is to maximize the change of opinions instead of making most people be influenced.

Fig. 6 shows the runtime of different algorithms. The confidence bound $\epsilon_i$ and the initial distribution of opinions do not influence the runtime of the algorithms. Therefore, we just show the runtime in different networks. The runtime of Greedy algorithm increases fast when $k$ increases. The runtime of heuristic algorithm is large although $k$ is small, nonetheless it increases very slowly when $k$ increases. The reason is that the cost of excluding some nodes is high. DegreeDiscount algorithm is a famous algorithm with low cost. Although the runtime of heuristic algorithm is always larger than that of DegreeDiscount algorithm, the distance between the runtime of the two algorithms is not very big if $k > 50$. Therefore, when $k$ is larger than 50, the runtime of the heuristic algorithm is acceptable.

6. Conclusions

In this paper, we study the issue of controlling the uncertain diffusion process in which the diffusion probability is influenced by evolutionary opinions. We present a model by extending the independent cascade model. In our model, each individual in social network has own opinion, and the probability with which he/she forwards the information is modified by the distance between his/her opinion and others’. Besides, individual may change his/her opinion by assimilating others’ opinions.

Based on theoretical analyses and simulations, we briefly discuss the properties of diffusion model and obtain some conclusions. For instance, if most nodes hold similar opinions initially, the initiator’s opinion can significantly influence
Fig. 4. \((|R| - |I|)/|N|\) with different selection of initiators are shown in this figure. The confidence level \(\epsilon_i = 0.5\) in these simulations. The initiator’s opinion is the most suitable value obtained in simulations.

(a) \(f_O(x) = f_i(x)\)  
(b) \(f_O(x) = f_{ii}(x)\)  
(c) \(f_O(x) = f_{iii}(x)\)

(4.1) Email network

Fig. 5. \((|R| - |I|)/|N|\) with different selection of initiators are shown in this figure. The confidence level \(\epsilon_i = 1\) in these simulations. The initiator’s opinion is the most suitable value obtained in simulations.

(a) \(f_O(x) = f_i(x)\)  
(b) \(f_O(x) = f_{ii}(x)\)  
(c) \(f_O(x) = f_{iii}(x)\)

(5.1) Email network

(a) \(f_O(x) = f_i(x)\)  
(b) \(f_O(x) = f_{ii}(x)\)  
(c) \(f_O(x) = f_{iii}(x)\)

(5.2) Facebook network
the number of people who forward the information. If nodes’ initial opinions are polarized initially, the influence of the initiator’s opinion will be limited. What is more, larger confidence bound usually lead to larger active rate.

Then, we present an optimization problem at the background of voting to show how to control the diffusion process. In detail, we model a voting mechanism in which individuals can choose one of the two candidates or abstention, and they vote with probabilities that depend on their opinions. In order to maximize the advantage of one candidate, we present theoretical analyses for the problem and develop strategy to influence individuals’ opinions based on the diffusion process. The results show that traditional influence maximization algorithms are not applicable to the problem, and our strategy can achieve the expected performance.

Our work and conclusions can present some inspirations to the real world applications. For instance, when the distribution of opinions is symmetrical, it is usually thought that the initiator’s most suitable opinion should support our faction. However, in the simulations, the most suitable value is less than zero when opinions are initially polarized. In this case, although consolidating the supporters of our faction is useful, reducing the supporters of hostile faction will be more effective. In addition, at the background of marketing, polarized opinion can be consider as mighty commending a product while slightly biased opinion can represent the euphemistic commendation. The conclusions in Section 5.4.1 show that when most people’s opinions are neutral and their confidence bound is small, slightly biased opinion is better than polarized opinion for the initiators. It means that when the consumers are not biased towards the product but suspicious, euphemistically commending the product may be better.

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